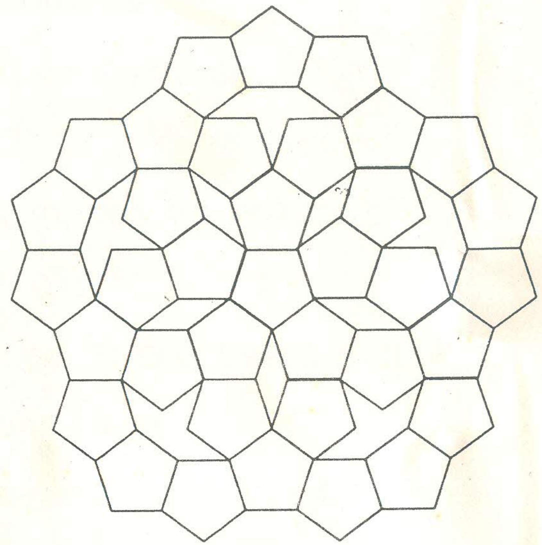
FIVEFOLD SYMMETRY IN 2D AND 3D

special project



by
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guide
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APPROVAL SHEET

The special project entitled "FIVEFOLD SYMMETRY IN 2D AND 3D" undertaken by Sunil William Moothedath is approved for the partial fulfillment of the requirements for the postgraduate degree in Industrial Design.

	Signature	Date
Guide		24.6.96.
Chairman		
Examiner		

ACKNOWLEDGMENTS

I would like to thank Prof . A.G.Rao for his continued support , encouragement, enlightening discourses and long walks and also everybody else for being here.

INTRODUCTION

This project entitled fivefoldness in 2D and 3D was taken up due to the special relevance it has that it is one of the most widely occuring form of symmetry in nature specially in the plant kingdom. In this project it was decided to look into the meaning of symmetry, the various aspects of expression of fivefold symmtery and its possible connection with weaving in bamboo.

The first part of the project involves mainly the documentation of symmetry and fivefoldness. The second part consists of exploration of bamboo weaving in 2D and its extrapolation into 3D.

The main aim of this project was to get a better understanding of symmetry specially fivefold symmetry and to open a new direction for exploration into bamboo weaving using fivefoldness in 2D and 3D

WHAT IS SYMMETRY ?

The concept of symmetry has two opposing aspects, one of transformation or change and the other of conservation or invariance. That which is conserved during a change is an invariant.

The set of transformations which keeps something invariant is its symmetry group. The theory of symmetry considers that all transformations of a system are executed at the level of a certain set of elements which are equivalent in some particular respect. The set of elements and their structural relationships forming the complete system are conserved as a single whole.

Different ways of distinguishing the structural sublevels associated with a particular object lead to different definitions of its symmetry groups. We therefore define symmetry as the law governing the constitution of structural objects or more precisely as the group of automorphisms conserving the qualitative completeness of the system under consideration. The concept of symmetry can also be defined as the state of invariance of a configuration of elements under a group of automorphic transformations.

The word symmetry is generally used to mean something like well proportioned, well balanced and also to denote that sort of concordance of several parts by which they integrate into a whole.

We define the symmetry group of an object as the highest possible group of automorphic transformations mapping any integral structural object, consisting of elements equivalent in the sense of relative equality, onto itself.

It can be noted that neither in man-made objects nor in nature (crystals, plants, animals etc.) is symmetry ever achieved with perfect mathematical precision.

TERMS IN SYMMETRY

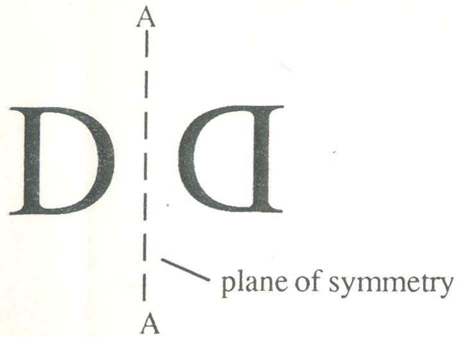


figure . 1

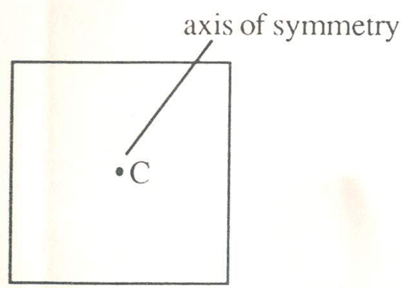


figure . 2

Planes of Symmetry

The imaginary plane which divides a figure into two mirror-image parts is called the symmetry plane. In figure 1 AA is the plane is symmetry. It is denoted by the letter m from the English word 'mirror'.

Symmetry Axis

A symmetry axis is a line normal to the figure such that when the figure rotates around it, the figure comes into coincidence with itself several times. In figure 2 the center of the square C is the axis of symmetry. The number of coincidences in one complete rotation is called the order of the axis. The smallest angle of rotation for which the figure coincides with itself is the elementary angle of rotation.

Relative Equality

The concept of relative equality is important to the theory of symmetry. The objects are called equal in relation to some particular feature if both objects possesses this feature. These features can be of orientation, physical properties, qualitative features etc.

We can note that nature does not and cannot allow absolute equality between two objects separated in space and time(perfect equality is possible only in a mathematical model), also that in real or relative equality it is essential to specify a criterion or more precisely to specify the measure of equality.

Geometric Regularity

We say that a thing is constructed in a geometrically regular manner if it may be divided without a remainder into equal parts with respect to a specific geometric feature. For example a square is geometrically regular in that it can be divided into four equal smaller squares without any remainder, similarly a circle may be considered as an infinite no of points equidistant from another point lying at the centre of the circle. Similarly an archimedian spiral is a geometrically regular figure since it consists of infinite number of points satisfying the equation $r = a0$

Symmetry is a special kind of geometric regularity in the sense that a symmetrical object consists of geometrically and physically equal parts appropriately disposed relative to one another. Appropriate disposition implies that the state of order should be identical in a specific sense for all the parts

Symmetry Element

A symmetry element of a figure is the geometric locus of points which remain in place when a specific symmetry transformation is effected or repeated any number of times. The set of such transformations form a cyclic group which we associate with the symmetry element.

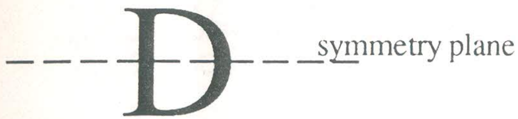


figure . 3 : mirror symmetry 2m

Mirror Symmetry

Any object which when after a symmetry transformation of reflection across a symmetry plane coincides with itself is said to exhibit mirror symmetry, ie. the object will be made of two mirror image parts (fig . 3). It is also called bilateral symmetry.

A majority of the plant and animal life on earth exhibit this type of symmetry. This kind of symmetry is encountered wherever by virtue of the living conditions of the species, the directions of up & down, forward & backward are essentially different while movements to the left and right are executed with the same frequency, here one of the chief functions are of rectilinear motion.

It is due to this fact that a majority of man made objects mimics the nature of bilateral symmetry which is seen around him.

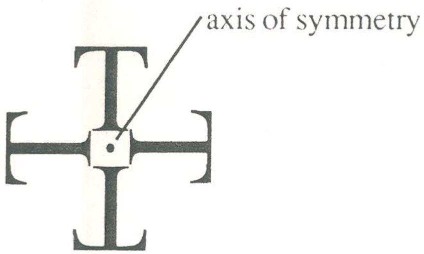


figure .4 : rotational symmetry order 4

Rotational Symmetry

An object which after a symmetry transformation of rotation around a symmetry axis coincides with itself is said to exhibit rotational symmetry. The order of symmetry is equivalent to the order of the axis. This kind of symmetry is also called axial symmetry or foldness. Hence an object that has an order of the axis as four will be said to have fourfold axial symmetry (fig . 4).

figure . 5 : translational symmetry

Translational Symmetry

An object which after a symmetry transformation of translation through space coincides with itself is said to exhibit translational symmetry. A figure which is invariant under a translation shows what in the art of ornaments is called " infinite rapport " ie. repetition in a regular spatial rhythm (fig .5).

A pattern invariant under the translation t is also invariant under its iterations $t^1, t^2, t^3, \ldots$ moreover under the identity $t^0 = I$, and under inverse t^{-1} of t and its iterations $t^{-1}, t^{-2}, t^{-3}, \ldots$ If t shifts by an amount a then t^n shifts by the amount na (n=1,+1,+2,.....)

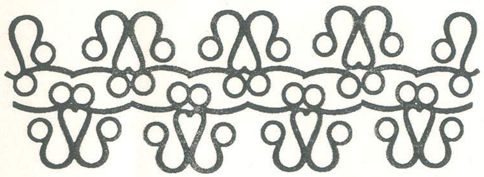


figure . 6 : ornamemtal symmetry

Ornamental Symmetry

It is a derivation of the translation symmetry with or without reflection symmetry (fig . 6). It is mainly used in arts as borders and reached its height of expressions in the baroque times when the ornamentation was give as much or even more elaboration than the object to be projected.

In case of one dimensional patterns only two type of symmetry patterns are possible.
(Ref -1: pg 48).

In case of one sided bands or patterns , there are seven symmetry classes.
(Ref - 2 : pg 92).

In case of two sided bands or patterns there are 31 symmetry classes.
(Ref - 2 : Pg 96)

In case of network patterns there are 17 symmetry classes . It comes to mean that there are only 17 ways of tiling for space filling possible.
(Ref- 2 : Pg 134).

In case of the symmetry elements in layers. There are 80 symmetry classes of layers. (Ref -2 : Pg 195).

In case of three dimensional spaces there are 230 space symmetry groups.
(Ref - 2 : Pg 207).

FIVEFOLD SYMMETRY

Any symmetry group which displays axial symmetry with order of the axis as five is considered under fivefold symmetry. Fivefold symmetry has the special significance that it has a wide expression commonly in flowers, fruits, molecules, logos, buildings, religions and mysticism.

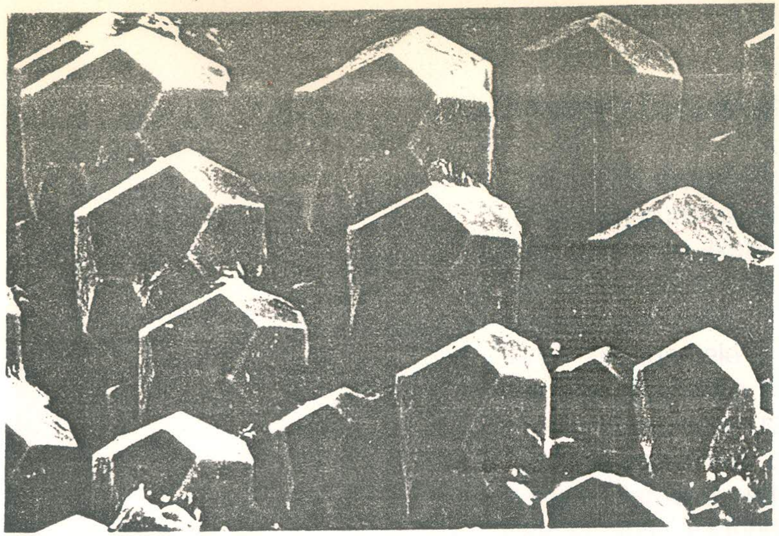
The fact has to be stated here that fivefold symmetry was considered as a forbidden symmetry in the science of crystals, specifically because fivefold symmetric objects cannot be stacked into a space filling pattern similar to how pentagons cannot be arranged to fill space in 2D.

This fact was accepted until 1984 when in a rather new state of matter called quasi crystalline state, for the first time five fold symmetric forms were observed. It was noted that the quasi crystalline form of Al-Cu-Ru had pentagonal dodecahedral growth patterns (fig.7) . Hence the field of science which most opposed the existence of fivefoldness in 3D now had to accept the fact of its existence . Even the tiling of pentagons in 2D was a problem until the discovery of the penrose pattern in 1974, the penrose tiling being an aperiodic tiling which gives regular gaps and hence is not completely space filling.

PLATONIC SOLIDS

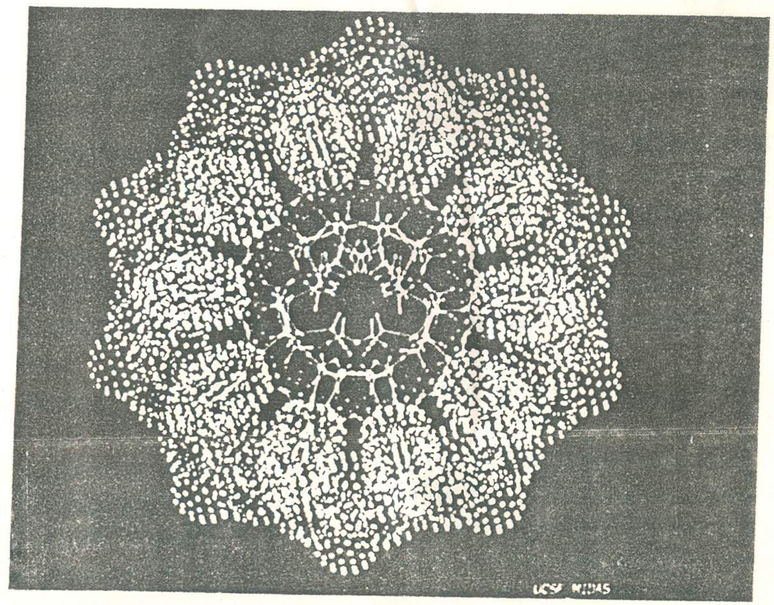
There are five regular polyhedrons namely - tetrahedron(4 faced), cube(6), octahedron(8), dodecahedron(12) and icosahedron(20). These five solids were known in almost all the ancient mystic circles. These platonic solid were given special significance and were regarded as symbols of the elements, the octahedron signifying air, cube-earth, terahedron- fire, dodecahedron-the universe and the icosahedron-water(fig .23).Of the five platonic solids, two namely the dodecahedron and the icosahedron have fivefold rotational axes. The point symmetry of both these bodies is the complete icosahedral group (2/m)35. Kepler had studied these two bodies in great detail, in his book *Mysteriun Cosmographicum* he connected the orbits of the six planets which were known at the time to a certain order of five regular bodies boxed inside each other.

In India the greatest and most powerful mandala the Shree Yantra or Shree Chakra has been ascribed a 3D form as an icosahedral crystal other than the popular Meeru form.



Pentagonal dodecahedron as growth morphology in quasicrystalline Al-Cu-Ru, synthesized by slow cooling from a melt. Specimen: Professor S. Politis, K.F.K. Karlsruhe. Scanning electron micrograph magnification 2200 times. Photograph courtesy of Professor Hans-Ude Nissen, ETH, Zurich, 1991.

figure : 7



A cross-sectional view of DNA showing a star decagon. (Photo by Robert Langridge of the Computer Graphics Laboratory of the University of California in San Francisco.)

figure : 8

FIVEFOLD SYMMETRY IN PHYSICS AND CHEMISTRY

The possibility of fivefold symmetry in crystals was born with the discovery of certain quasi-crystals which existed with a dodecahedral structure(fig .7). This had helped to explain many physical and chemical properties which were based on the crystal structure. The structure of the crystals is important when study with tensors are done.

In the field of astronomy the dodecahedral model can be used as a guide for star positioning in place of a cube.

The recent revolutionary results in chemistry include the discovery of a new allotropic form of carbon molecule in its pure state, other than the popularly known diamond and graphite. The structure of the new forms of carbon ie. C_{60} and C_{70} mainly found in space were actually modelled with the help of the geodesic model of R.Buckminster Fuller. The molecule came to be known as buckminsterfullerene having fivefold symmetry axes. The appearance of fivefold symmetry in organic chemistry has been common. Molecules like the $C_{20}H_{20}$ has the structure of a pentagonal dodecahedron.

FIVEFOLD SYMMETRY IN BIOLOGY

The world of living organisms find a large outlet for expression in fivefoldness. Flowers with five petals, starfishes, microorganisms(fig 13), viruses(fig 12), seeds, leaf growth, etc., are all marked examples of this phenomenon. Even the DNA has a decagonal cross section with pentose sugars towards the centre(fig 8). Also the famous HTLV-2 or AIDS virus has an icosahedral structure.

The angle of the golden section γ is very important in the spiral growth seen in plants (fig. 9)

Where γ is defined as $\tau = 360°/ (360° - \gamma)$ from which it follows that $\gamma = 137.51°$

This kind of spiral growth is sometimes referred to as Fibonacci growth. Such spiral growths are seen in leaves, petals, radiolaries(fig 10,11), pine cones etc.. The benefit of plant growth having leaves fall according to the Fibonacci growth is that the leaves are never directly above one another and hence the is possibility of getting maximum sunlight for photosynthesis(fig . 9).

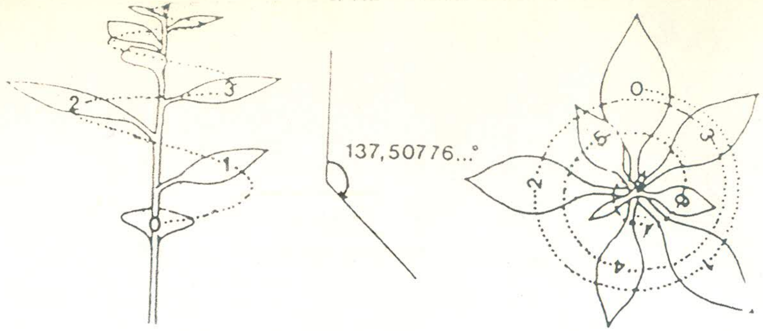
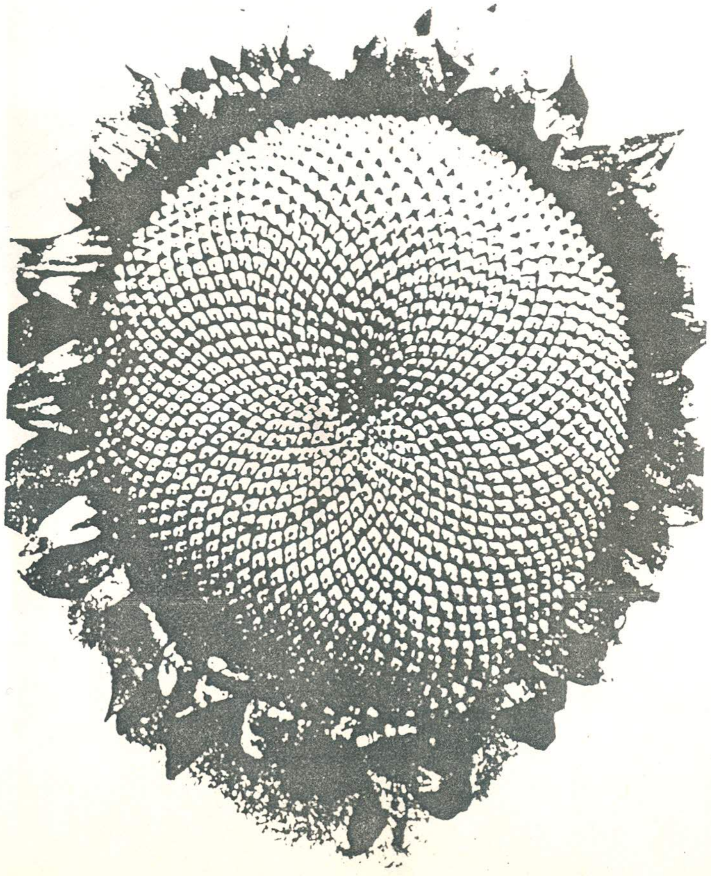


figure : 9



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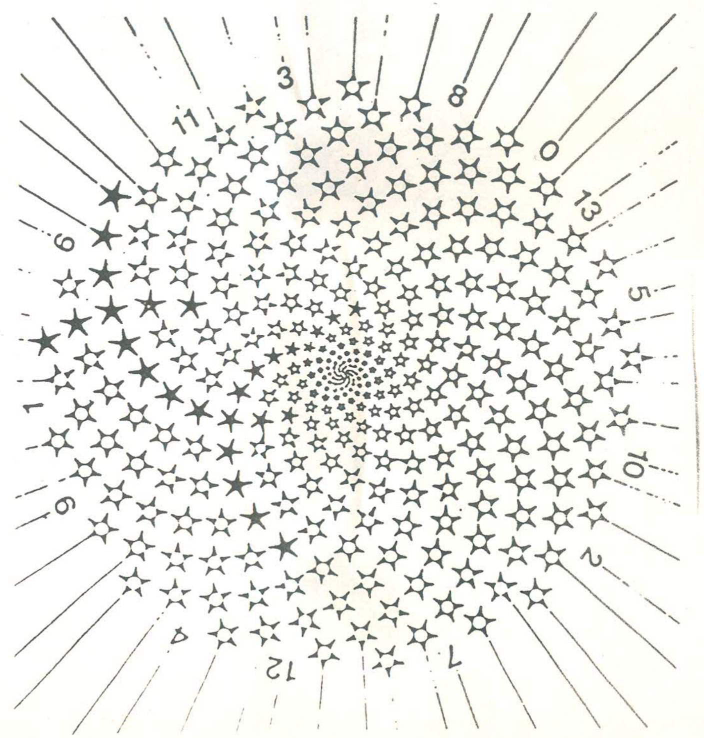

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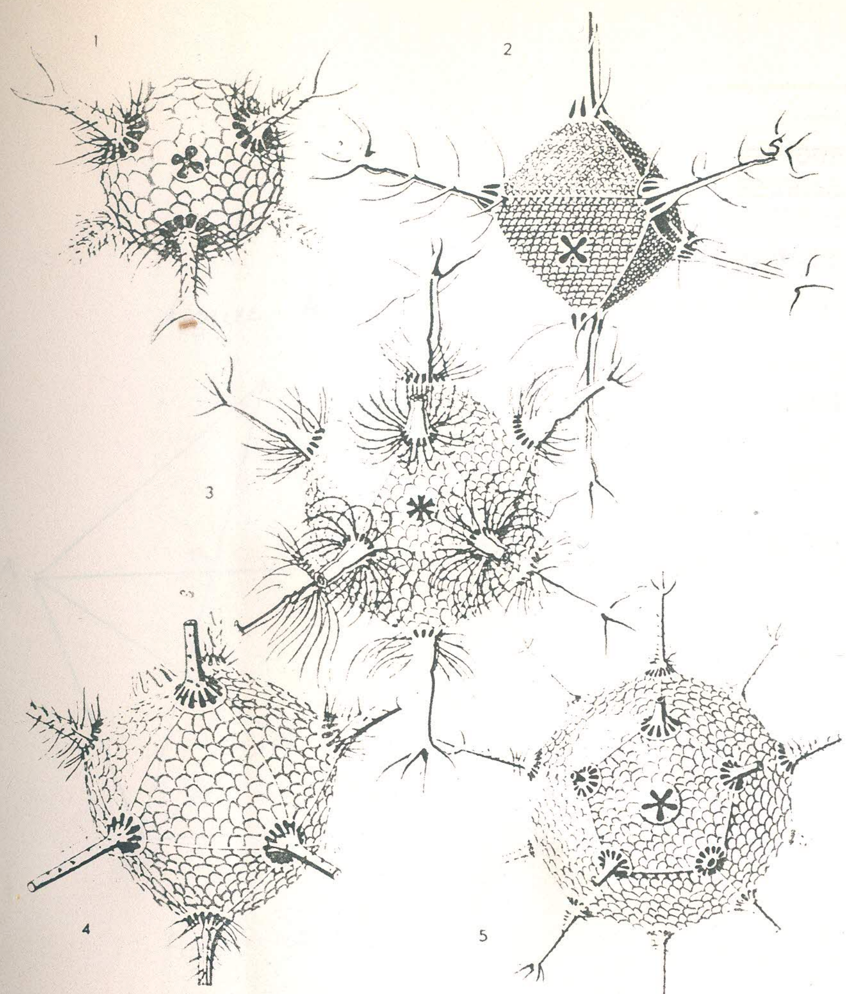

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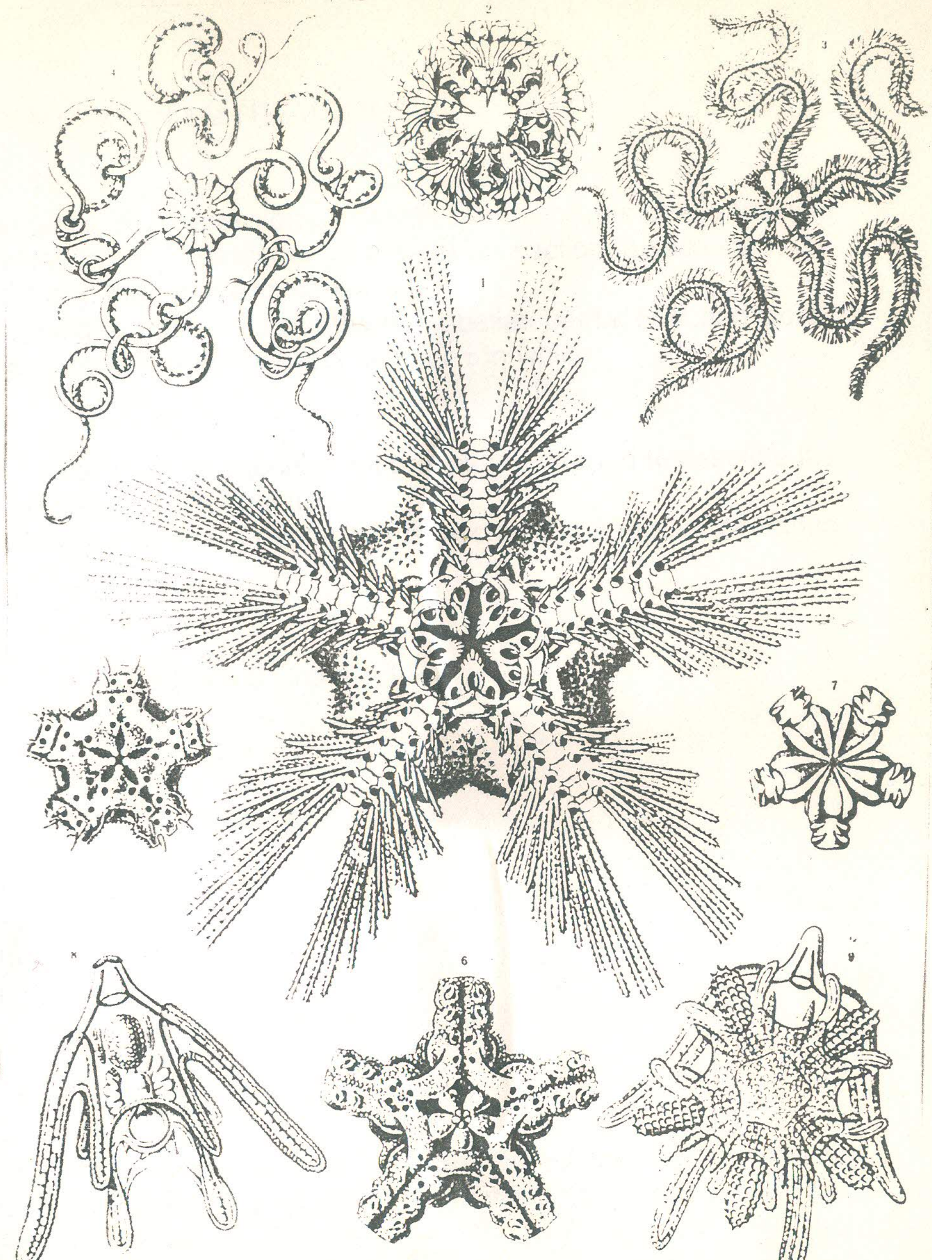
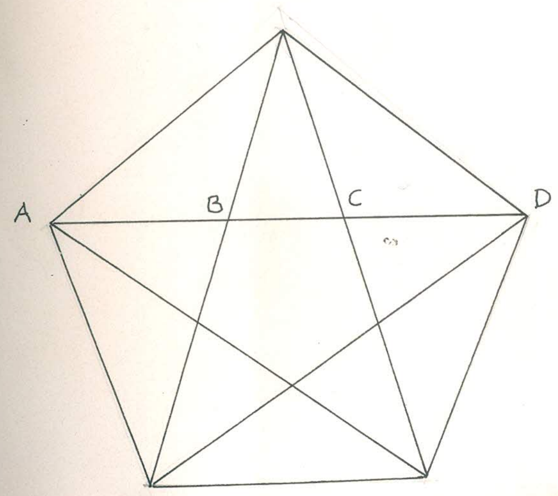


figure : 13

FIVEFOLD SYMMETRY IN MATHEMATICS

figure : A



In mathematics the expression of fivefold symmetry has been mainly in the form of the relation of the pentagon to the golden section, golden ratio, the Fibonacci number and the Fibonacci series.

It is interesting to note that both the non-abelian pentagonal and icosahedral point groups contain the Fibonacci number τ in which

$$\tau = \frac{5+1}{2} \quad .... \quad 1$$

It is to be noted that seven pentagonal point groups are needed to describe the symmetry m with fivefold rotational axes.

From the figure A where a pentagram is inscribed in a regular pentagon we see that

$$a = 36^\circ$$

Also we have

$$4\text{Cos } 2a = 2 \text{ Cos } a + 1 = 0$$

where

$$\text{Cos } a = \frac{5+1}{4} \quad .... \quad 2$$

Then from equations 1 and 2 we have

$$\text{Cos } 36^\circ = \tau / 2$$

we also have the equation

$$\tau^2 - \tau - 1 = 0$$

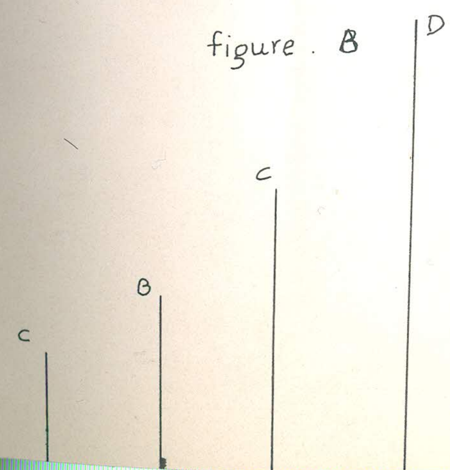
with the positive root

$$\tau = \frac{5+1}{2} = 1.6180339887$$

Fibonacci, alias Filius Bonaccio or Leonardo di Pisa found an interesting sequence of numbers named after him as the Fibonacci series which is as follows

0,1,1,2,3,5,8,13,21,34,55,........

figure . B



this series is also described as a recursive number sequence , such that any number in the series is equal to the sum of the previous two numbers. It is also noted that the ratio of any number in the series with its immediately prior number is equal to the Fibonacci number τ (this is true and more accurate for number higher up in the series).

It is seen from figure A and B that

BC , AB , AC , AD form a Fibonacci series

The Fibonacci number obeys various other interesting relationships in the number theory like

$$t^n = t^{n-1} + t^{n-2}$$

where n can be a positive or negative number.

$$\tau = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}$$

sometimes τ is also referred to as the most irrational number

$$\tau = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 \dots}}}}}$$

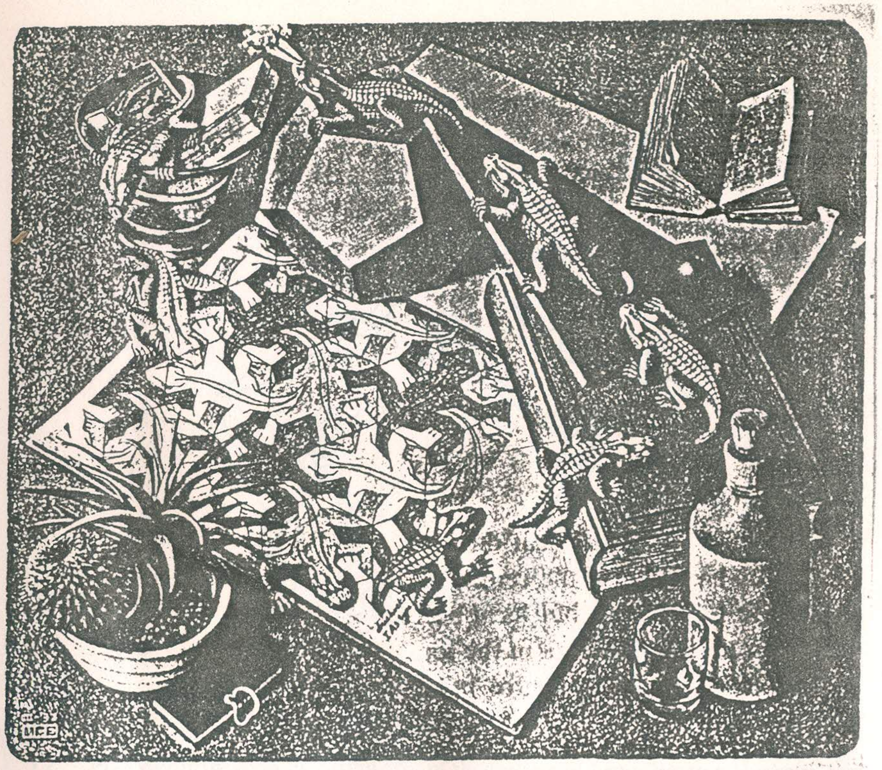


figure : 14 Reptiles (1943). Lithograph, 33.4 cm high × 38.6 cm wide. © 1989 M. C. Escher Heirs/Cordon Art-Baarn-Holland.

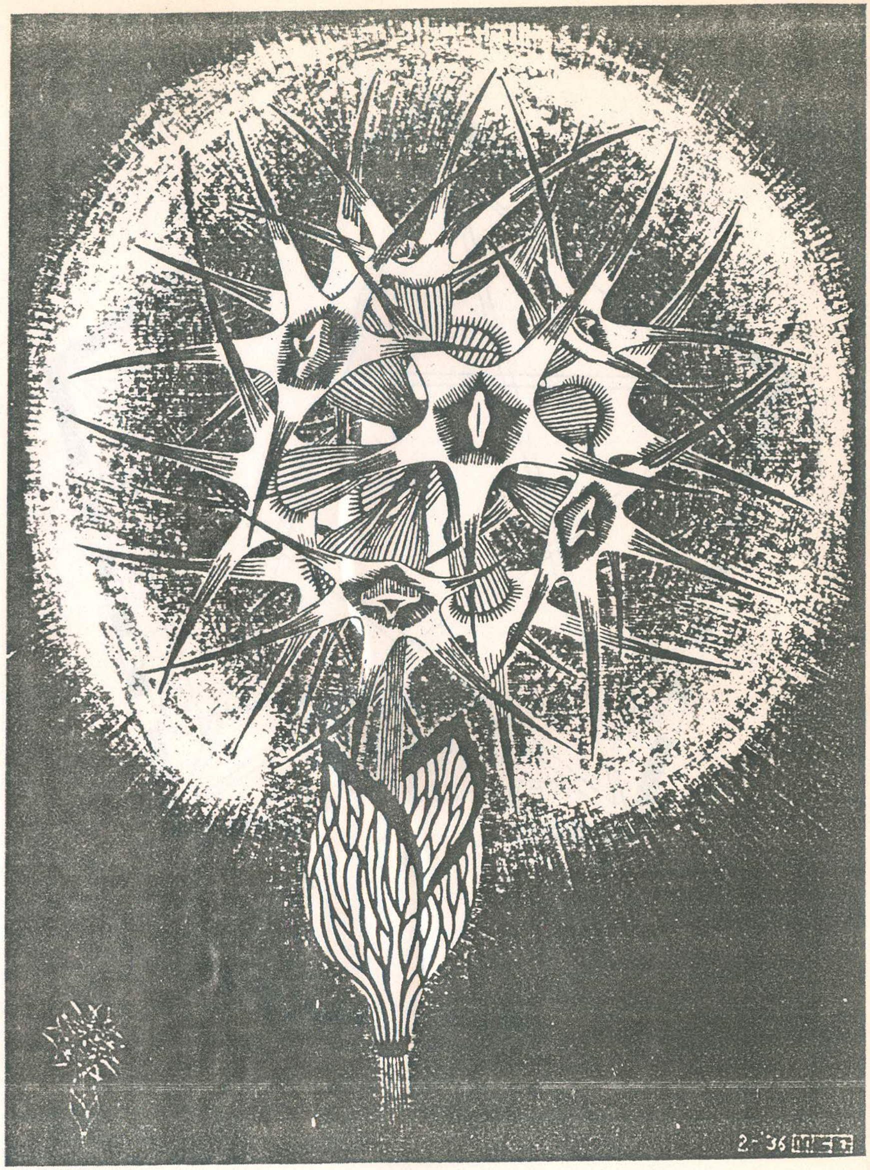


figure : 15 Prickly Flower (1936). Wood engraving, 27.7 cm high × 20.8 cm wide. © 1989 M. C. Escher Heirs/Cordon Art-Baarn-Holland.

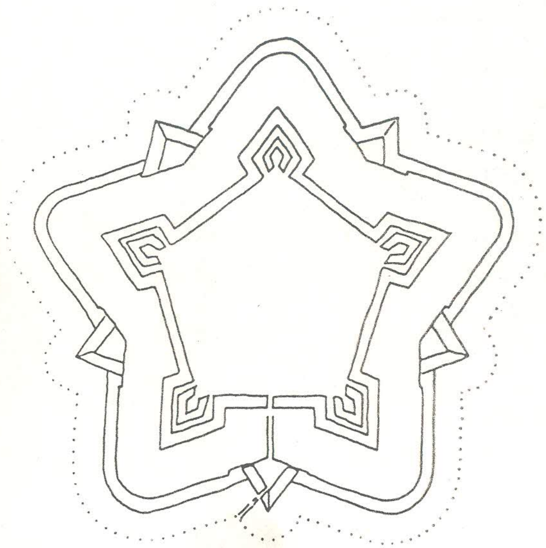


figure : 16 Plan of fortification by earthworks of the fortress of Lenti, Western Hungary, from the 17th century

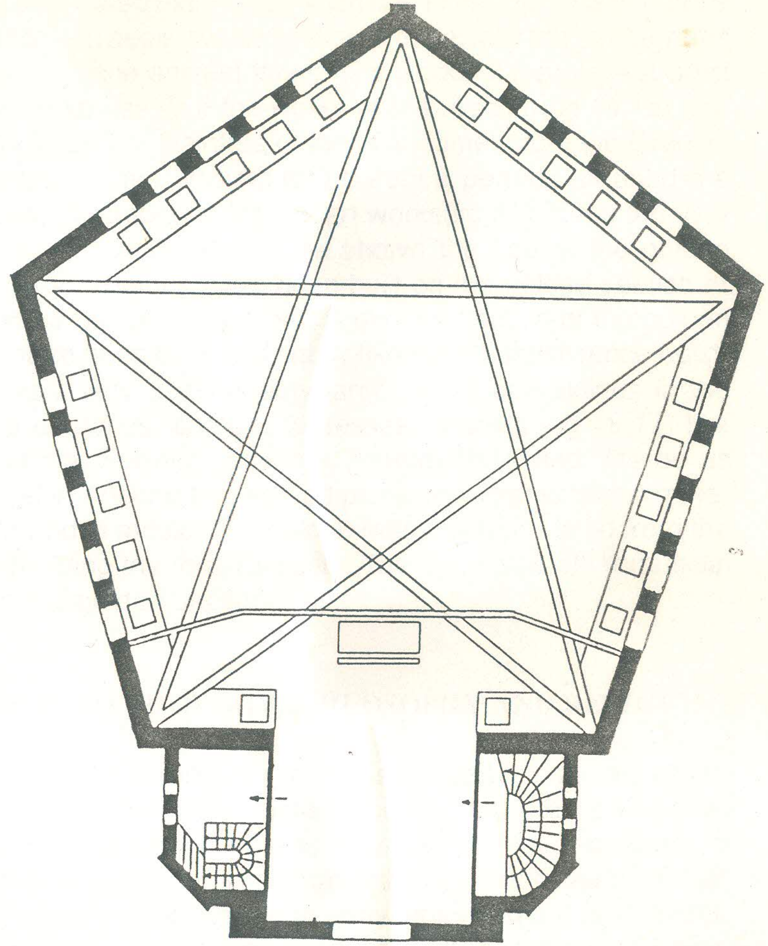


figure : 17 Overview plan of the Honvéd Square Calvinst Church made by József Borsos, architect of the church (from Ref. 3). The pentagram shows the five main arched rib-console which hold the ceiling.

FIVEFOLD SYMMETRY IN ART, ARCHITECTURE AND MUSIC

Fivefoldness has been used extensively in art due to its nearness to natural floral patterns. In ornament and seals fivefold patterns especially the pentagrams have been used extensively since ancient times. The works of many great artist show great exploits of fivefoldness, like the works of M.C.Escher(fig 14,15) and other artists like Salvador Dali. The painters of the olden times from the Greeks, the romans, the renaissance, the classical and baroque periods all used the magic of the golden section in proportioning the art works(fig 21). Even the very famous work of Leonardo di Vinci's *Mona Lisa* shown the use of the golden section.In architecture many buildings have been built on the golden section or golden ratio. The *Le Modular* by Le Corbusier is a direct reflection of the golden section. Some buildings have been built less subtly like the US intelligence building the Pentagon, making a very good enemy target. A lot of buildings in the medieval times including churches, castles, fortresses, citadels (fig 16,17) like those at Antwerp, Capsarola, Ferrana, Modena, Parma, Paris and Viterbo as well as the Plague hospital in Ancone has a good plan involving fivefold symmetry. Fivefoldness is also found in music. The golden ratio has been used from the times of Pythogoras for deciding the musical scale. There are also other musical scales built comletely on the golden section.

FIVEFOLD SYMMETRY IN MYTHOLOGY, PHILOSOPHY AND RELIGION

The pentagon has been a symbol of great mystical value in almost all the ancient cultures(fig 22). It has often been looked upon as a protection against evil . In Gothe's "Faust" Memphistopheles was exorcised with a pentagram. It may also be the reason that a quarter of all the national flags in the world contain the pentagram. It is found even on the American army tanks and the Soviet Star. In Christian beliefs, fivefoldness was given a new meaning as a symbol of Christ's crucifix with five wounds.

In Hindu mythology and religion there are many expressions of fivefoldness. Much of the Rig Veda is rooted in fivefoldness for example the elements of nature, the essences in man, the directions in space, the stages of life etc are all depicted in fivefold. The angle of the pentagon had special significance as an indication of reincarnation during the time of ancient Egyptians even in the later Christian era (fig . 18,19,20).

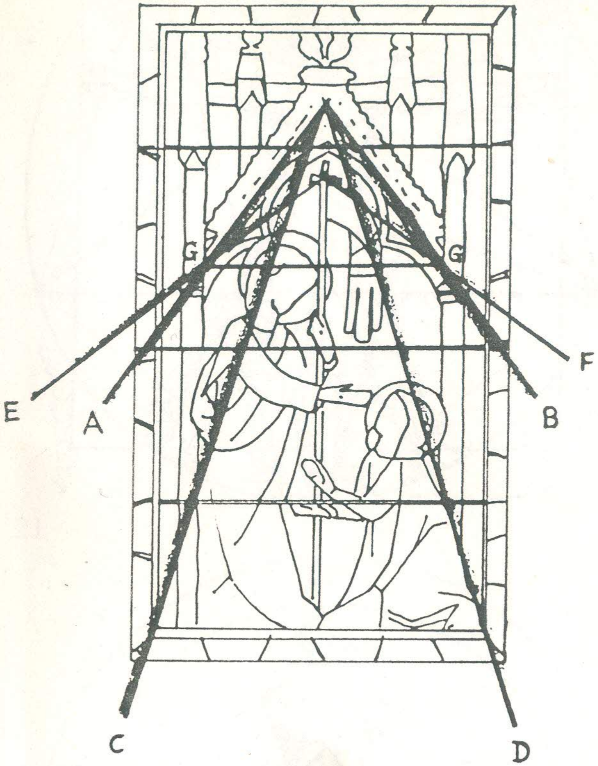


figure : 18 The angle AB of the vertex of the temple's roof has 72 degrees, the number of "descent" and "dispersion" of the Word. The angle CD has 36 degrees, corresponding to the nucleus of the whole symbolic pentagonal representation of the apparition of the resurrected Christ to the Virgin. The number of resurrection is hidden in the angle EF having its vertex in the center of the *labarum's* cross and connecting it with points GG. In this image resurrection through the cross is linked to the basis of the temple's triangle.

De la resurrectiõ d nre seigneur ihũcrist

figure : 19 Wood-cut from *La Légende Dorée* of Jacques de Voragine, Lyon, 1483 (108° angle stressed).

figure : 20 Resurrection of Jesus Christ."Rosarium Philosophorum," Frankfurt, 1550. (The 108° angle is stressed by bold line.)

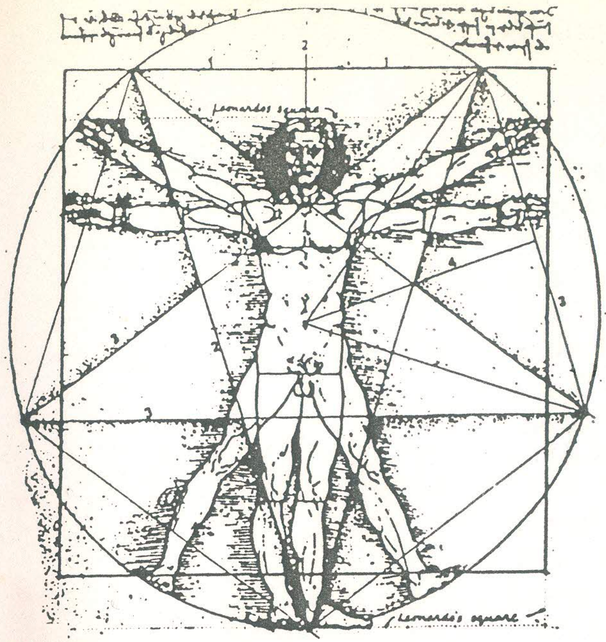


figure : 21

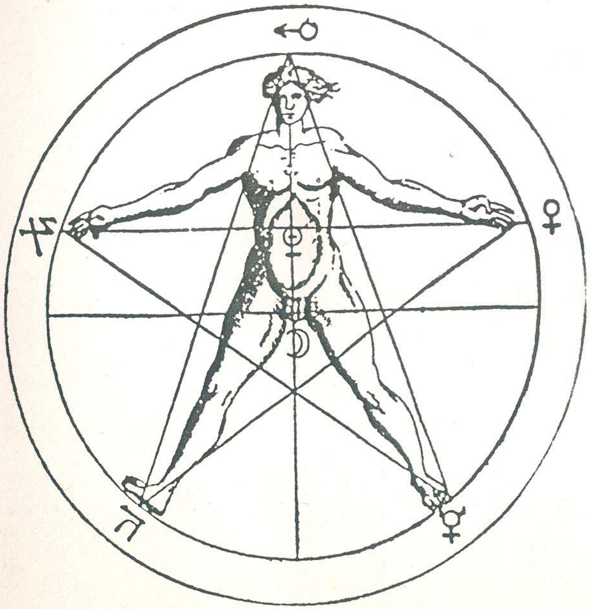
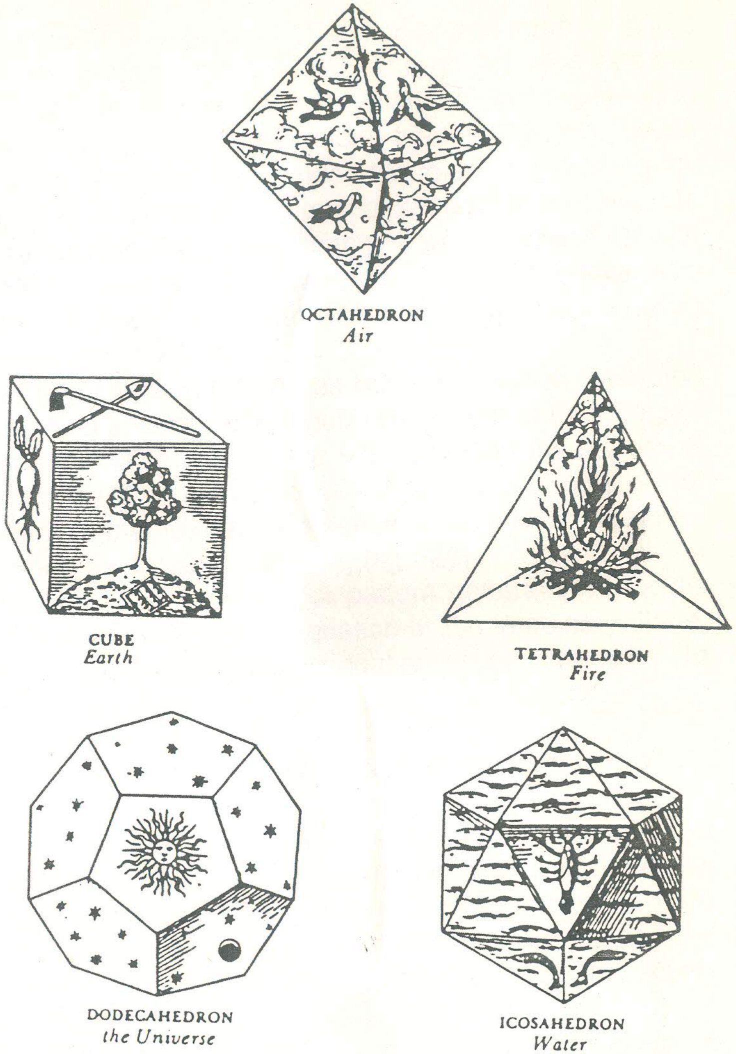


figure : 22

Illustration of the "Magic of Arbatel" by Cornelius Agrippa von Nettesheim (1486–1535).



The Platonic solids representing the four classical elements of the ancient world with the Dodecahedron as the cosmos.

figure : 23

PENTAGONAL TILING AND FRACTALS

We note that the length of the diagonal of a regular pentagon isτ times the side length, where t is the golden ratio 1.61803. The golden ratio has the property that $\tau : 1 = 1 : \tau - 1$. We see that the two diagonals trisect the pentagon into two obtuse golden triangles and one acute golden triangle (fig .24). These golden triangles can be further dissected into golden triangles or into a regular small pentagon and golden triangles. In this fashion if a pentagonal area were to be dissected into smaller pentagons and golden triangles we would end up with a tiling which is an ordered monomorphic asymptotic tiling based on regular pentagons (fig . 25). Such a pattern would be a source for pattern generation giving a rich foreground background composition.

Another form of pentagonal tiling which has been very widely accepted as the most compact space filling possible with regular pentagons in the Penrose Tiling. In this kind of pentagonal tiling spaces are left out inside the pattern of pentagonal tiles which cannot be filled up. But what is interesting to note is that these spaces follow a pattern in themselves and hence gives a visual feature . The penrose tiling is described as an aperiodic tiling (fig . 26).

Similarly fractals can be used as maps for pattern generation. Fractals are basically a plot of an iterative or recursive equation in the imaginary plane. Fractals with fivefoldness can be created using equation of the fifth order. To create such fractal patterns an equation of the form

$$\zeta \; — \; \zeta^{5} + m$$

is taken where m is a constant and ζ is a complex value of the form a + ib where i = -1

Once the integer points are selected each iteration represents a step along a path that hop from one complex number ζ to the next. The collection of all such points along a path constitutes an orbit. The basic goal is to understand the ultimate fate of all orbits for a given system. In the fractal output obtained the small irregular shaped regions correspond to those ζ which explode very slowly or whose trajectories are bounded.

Similarly other methods of solving equations like the Halley's method or the Newton's method would also yield interesting fractal outputs. These are useful as an artistic output which have a very high degree of intricacy and the feeling of sophistication.

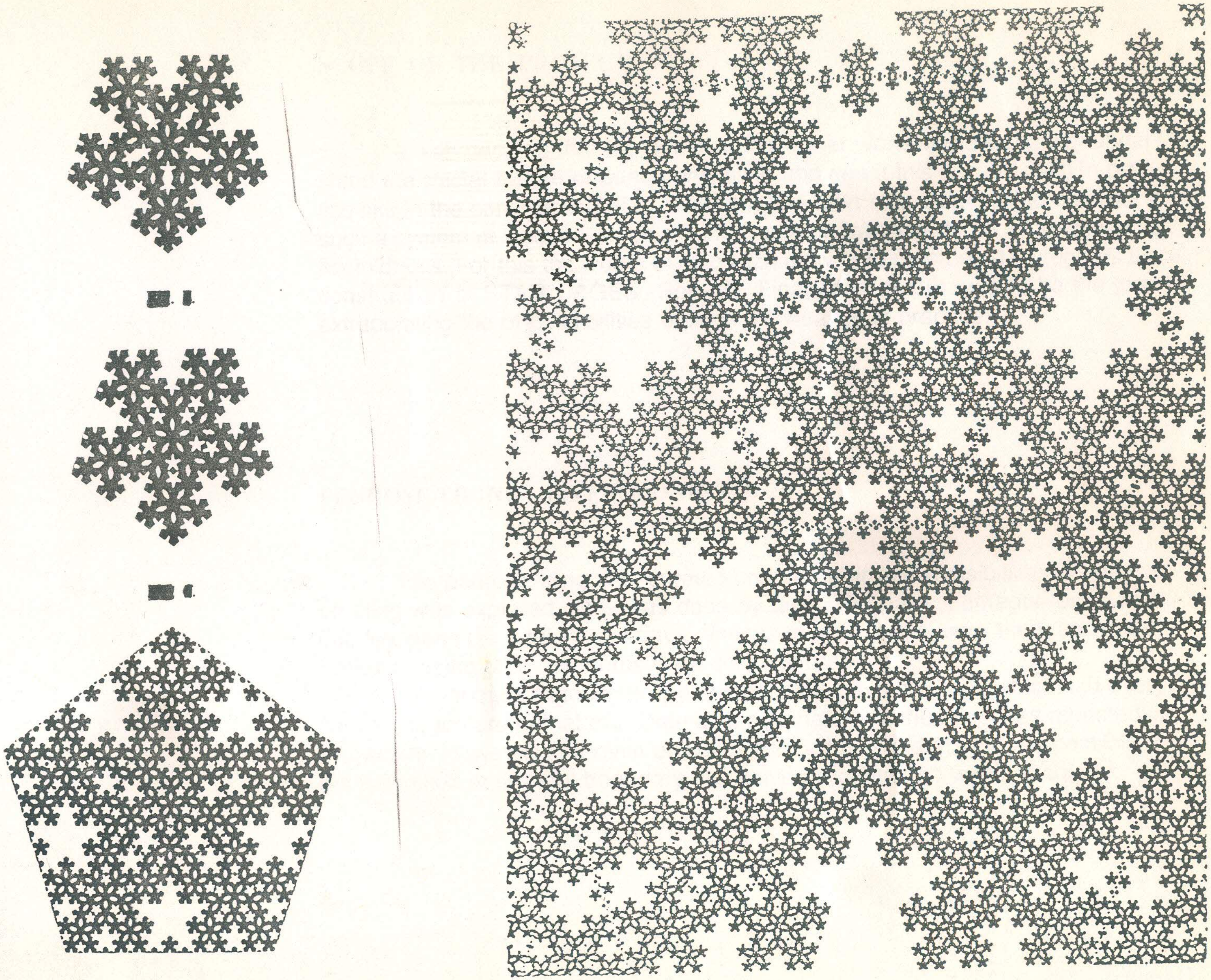


figure : 25

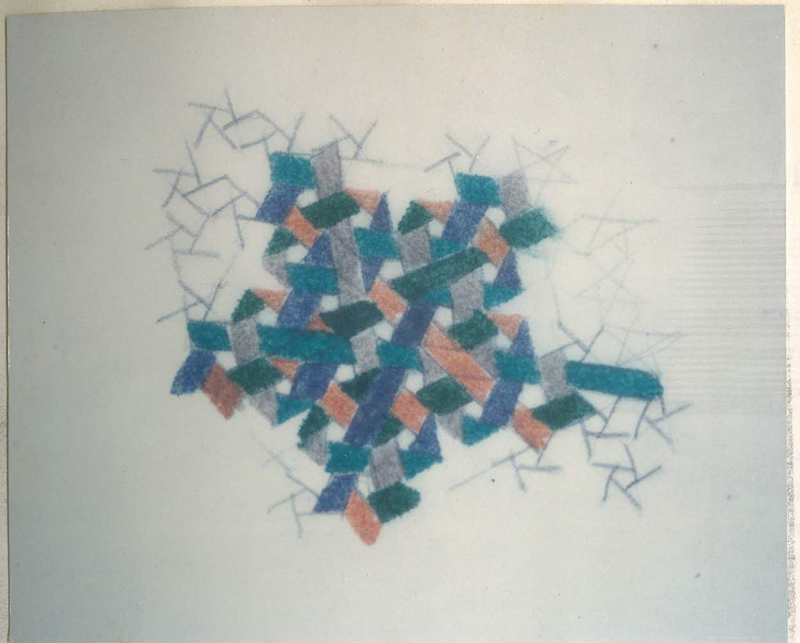
SCOPE OF THE PROJECT

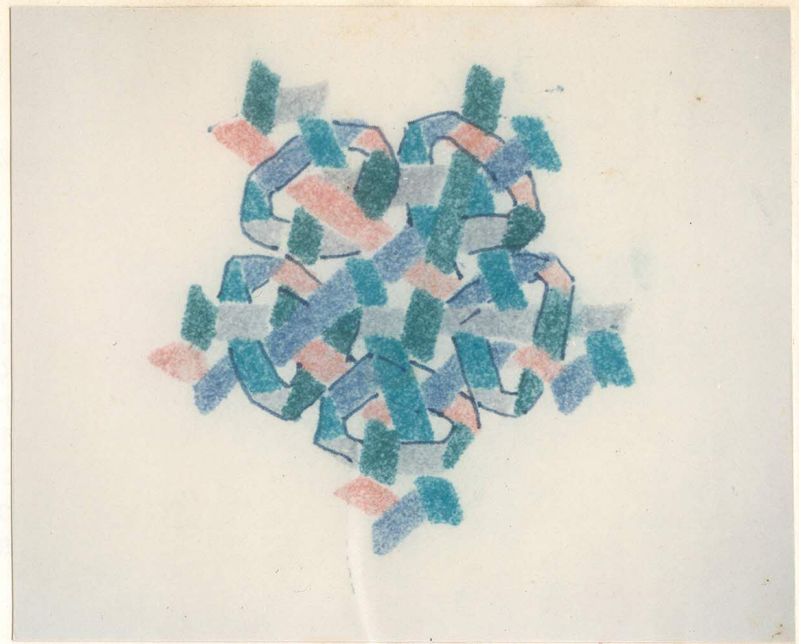
It was decided that the direction for further work would be to first understand the fractal nature of pentagonal tiling and see if this can be used in practice like in the construction of 2D weaves. The next step would be to consider if such a system of weaves which are in 2D can be translated into 3D retaining its fivefoldness. For this the works of Akio Hizume was to be studied as regards the construction of STAR CAGES (Ref - 4). Finally the output was to be the form extrapolating the of possibilities of such 3D weaves in practice.

PENROSE TILING - FIVEFOLDNESS IN 2D

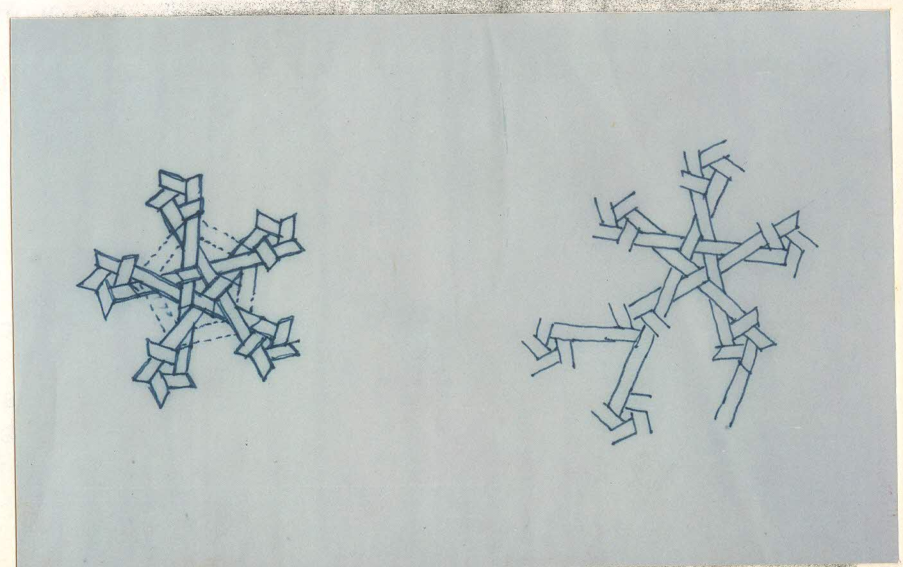
The penrose tiling was created on computer and possibilities of weaves on tiling was explored. This was done by using the basic pentagon as divided into five parts each taking a weave. Various combinations were tried, and possibilities of using other elements were also explored.

Here the pentagonal tiling of the penrose gave a weave which had avery interesting feature in that the same aperiodic nature of the tiling was repeated in the weave. However that main dawback noticed was that it was not possible to get a levelled or straight boundary that could be contained within a square.





2D WEAVES USING THE PENROSE TILING



POSSIBLE WEAVES AND COMBINATION OF EXTRA ELEMENTS

STUDY OF STAR CAGES

The method for the construction of star cages were explored and the star cages were constructed in both the TO-MAGARI and MU- MAGARI styles. The star cages were made with 30 sticks and also explorations were done on systems with upto 120 sticks. Star cages were also made in different materials like metal, coconut leaf fibre, bamboo and reads.

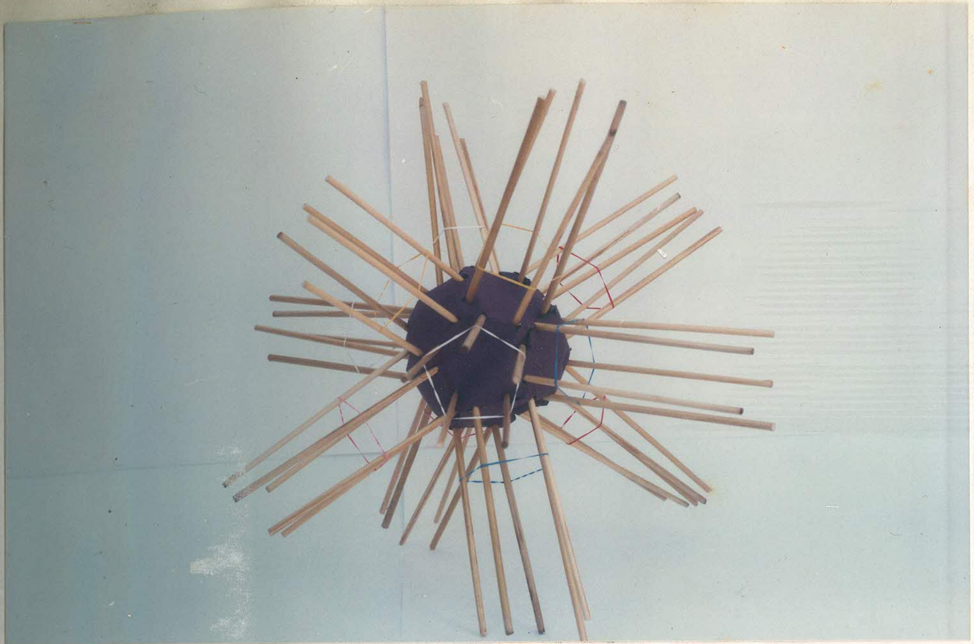
The constructions were mainly done with the basic pentagonal polyhedrons, the dodecahedron and icosahedron. The models were held using twines or rubber bands. These star cages are supposed to be self supporting structures.

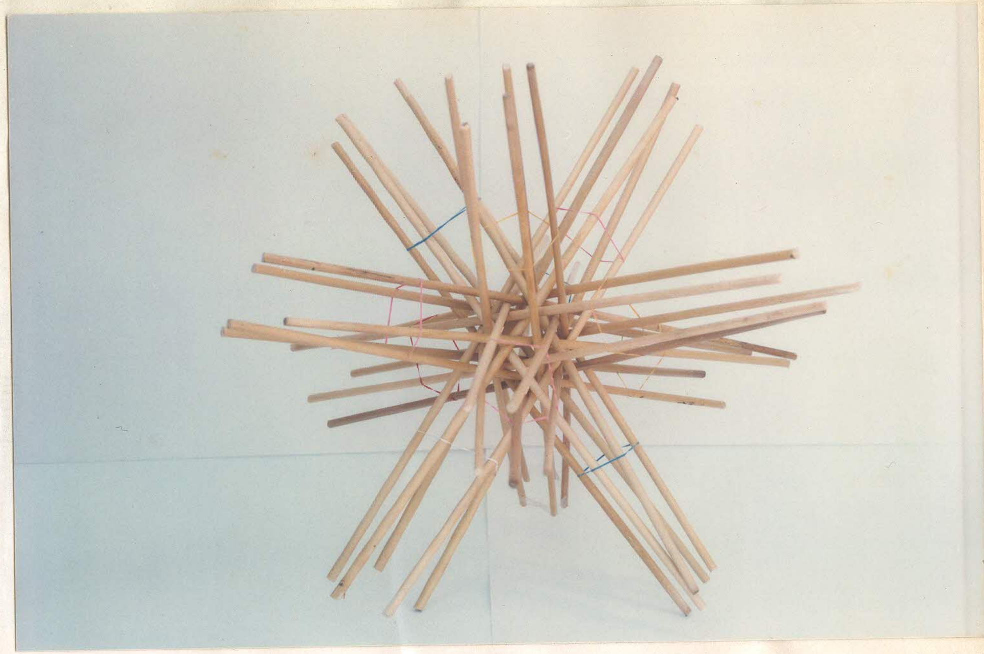
MU - MAGARI

This was constructed using the basic polyhedron, the dodecahedron on whose pentagonal faces other pentagons were draws so that the newly drawn pentagon was skewed by an angle of 18°. This was done so that when the sticks were passed through the vertices of the drawn triangles they do not intersect within the dodecahedron. In the minimal case we would obtain a star cage with 30 sticks. If the number of the sticks are to be increased we will have to have the vertices in the form of the penrose lattice, which then would give us a 3D penrose lattice.

TO - MAGARI

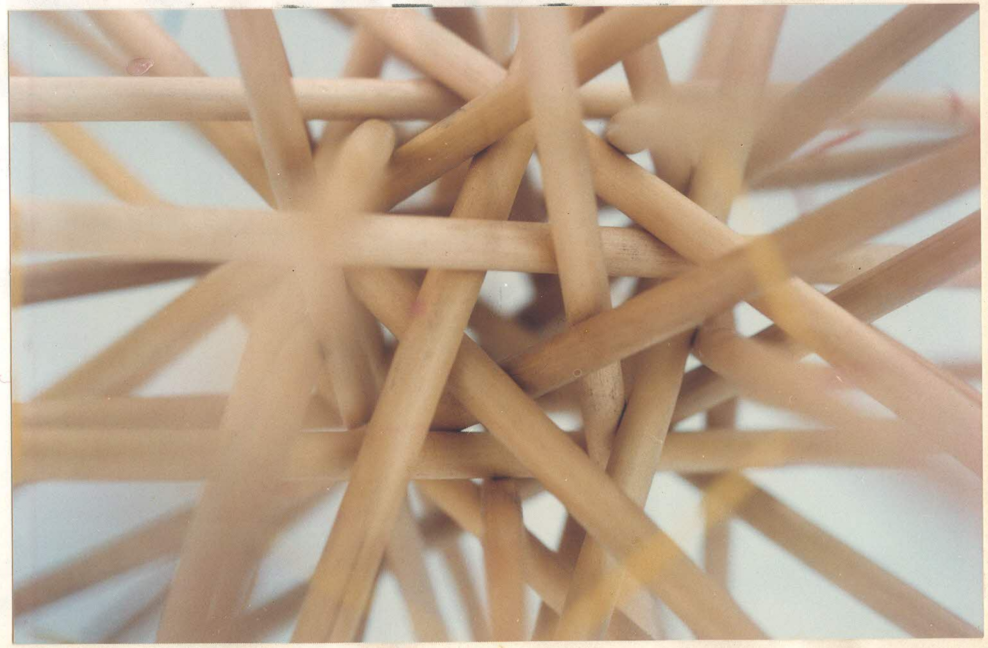
This was constructed with the basic polyhedron, the icosahedron on whose triangular faces other triangles were drawn with its vertices skewed at an angle of 12° . This again helps to get maximum distance between vertices and parallel edges of the drawn triangles. Here also the minimal case is with 30 sticks and for greater number of sticks would require a triangular tiling.



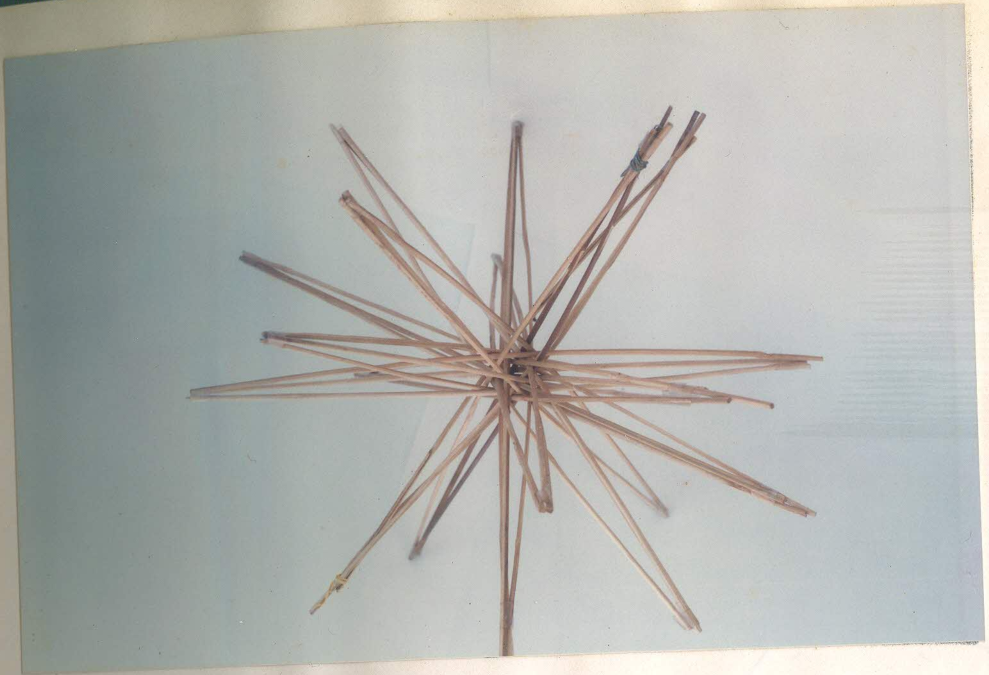


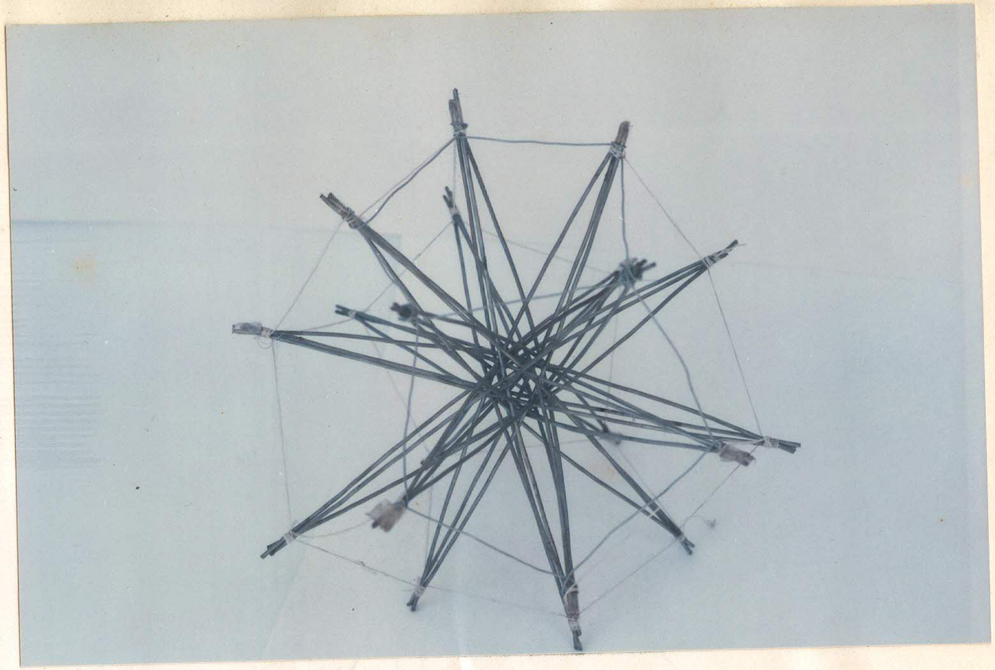
THE MU-MAGARI -

THE MU-MAGARI
WITH DODECAHEDRAL CENTRAL STRUCTURE



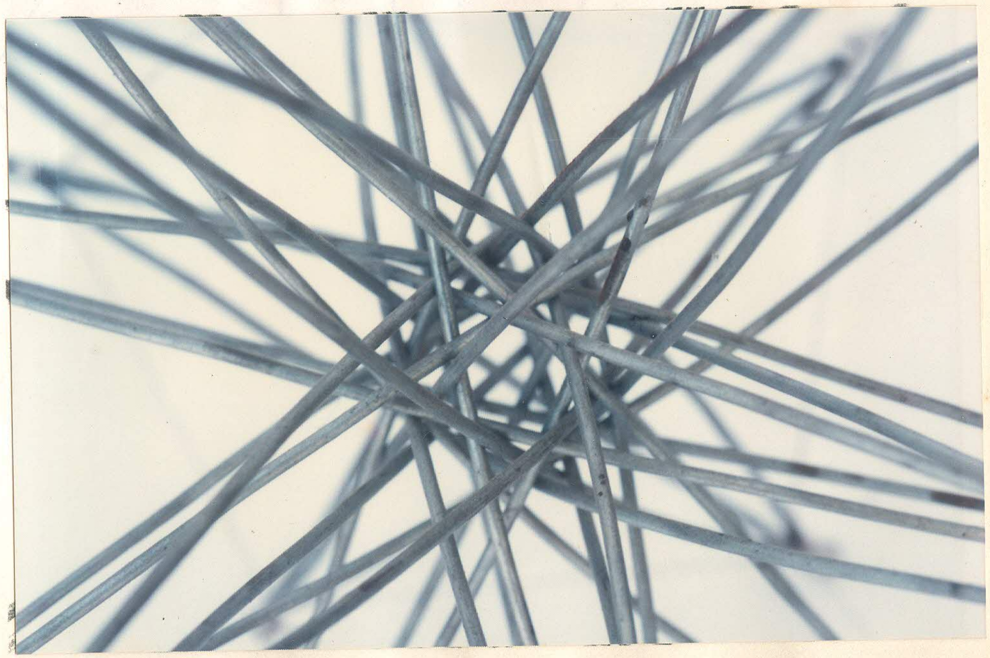
MU-MAGARI - CLOSE UP OF CORE



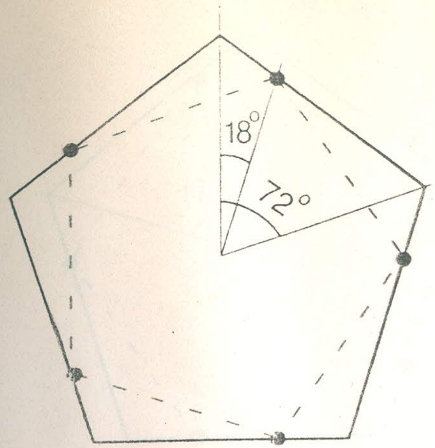


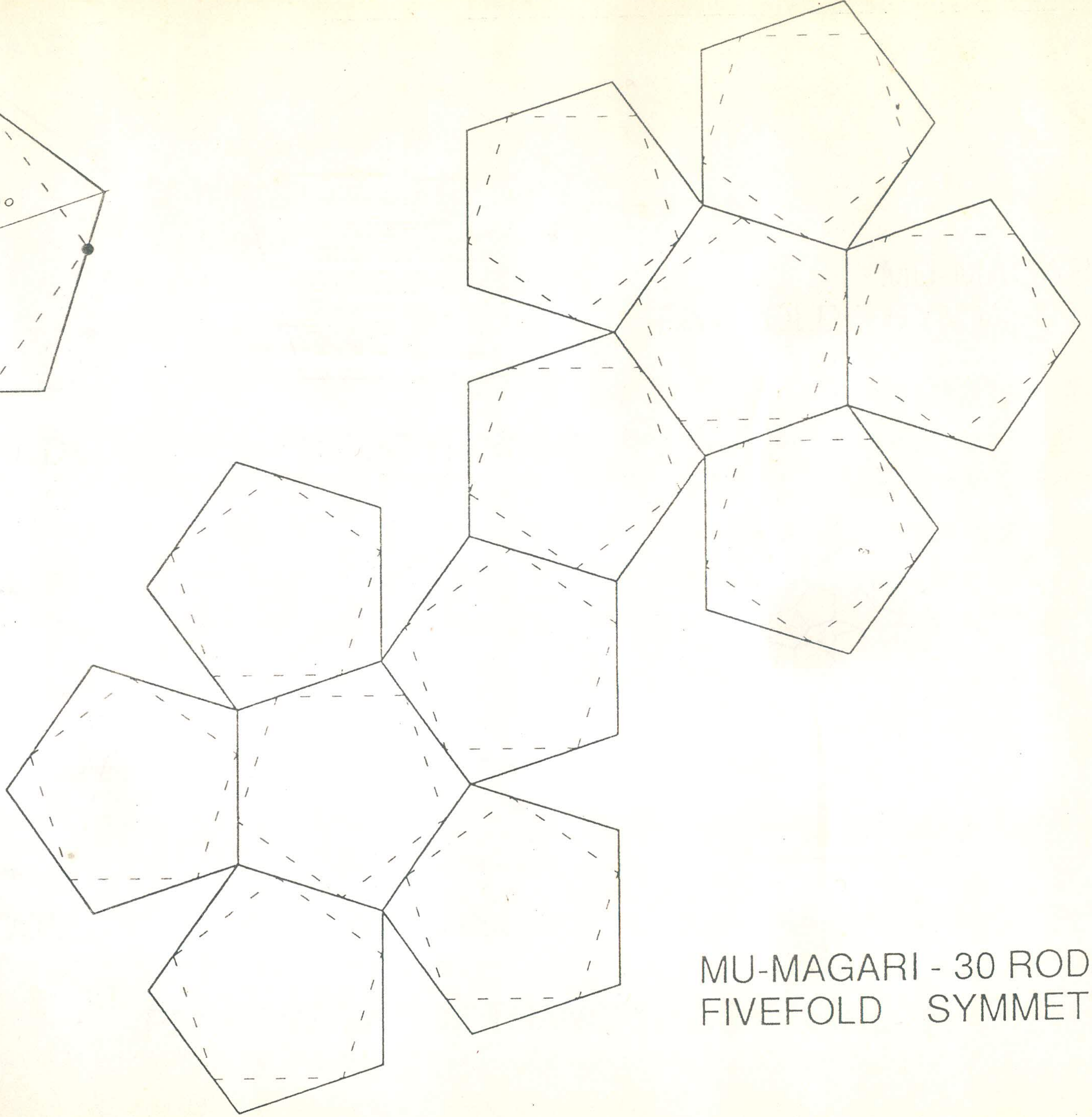
THE TO-MAGARI WITH ENDS CONNECTED
FORMING A DODECAHEDRAL FRAME

THE TO-MAGARI

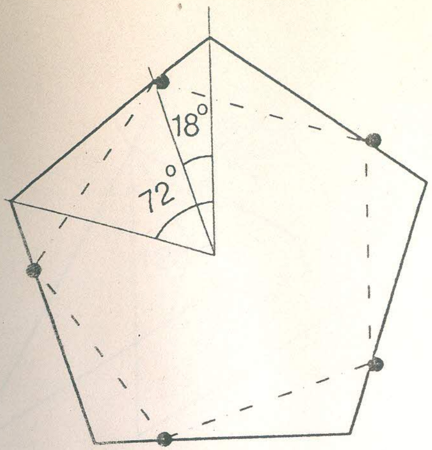


TO-MAGARI - CLOSE UP

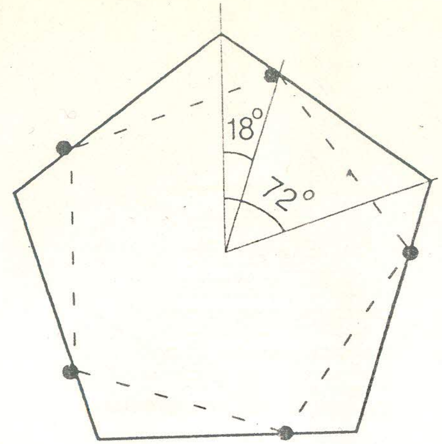




MU-MAGARI - 30 RODS
FIVEFOLD SYMMETRY



LEFT HANDED



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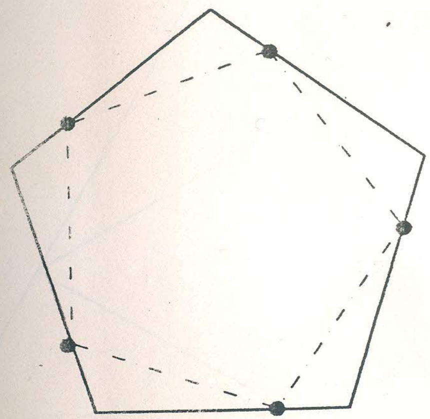
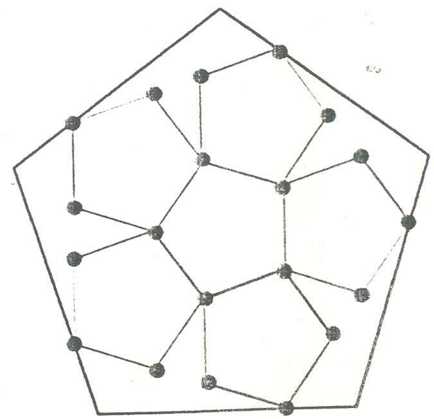
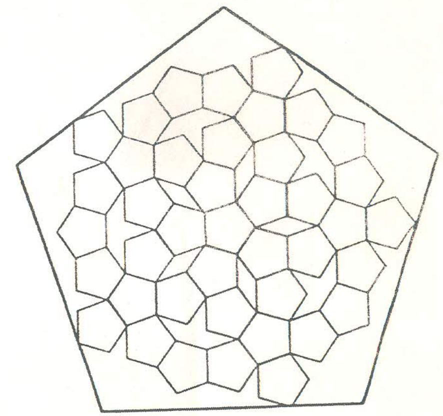
FIVEFOLD MU-MAGARI SYMMETRY



30 RODS



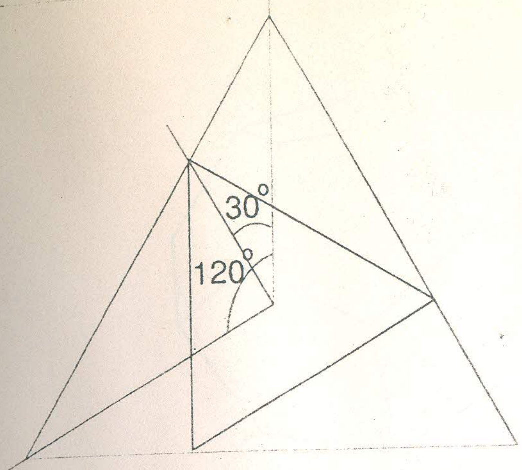
120 RODS



600 RODS

PENROSE LATICE TRANSPOSED IN 3D

FIVEFOLD TO-MAGARI SYMMETRY



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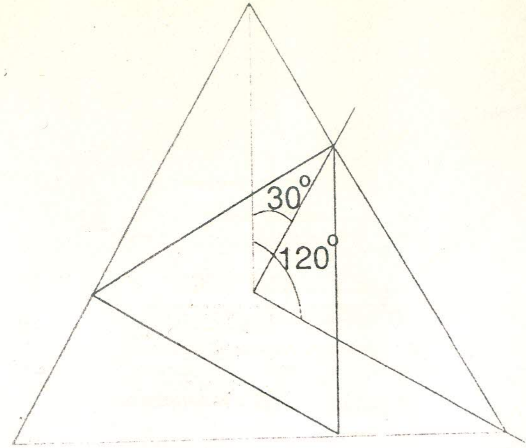

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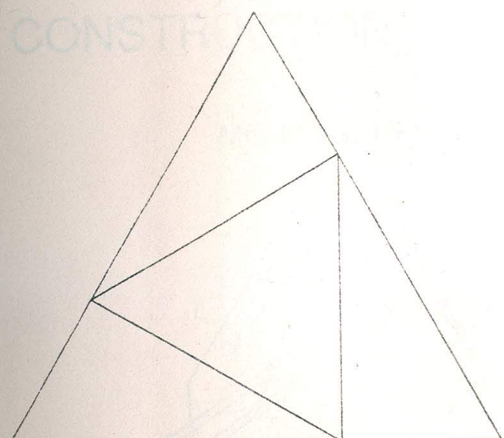
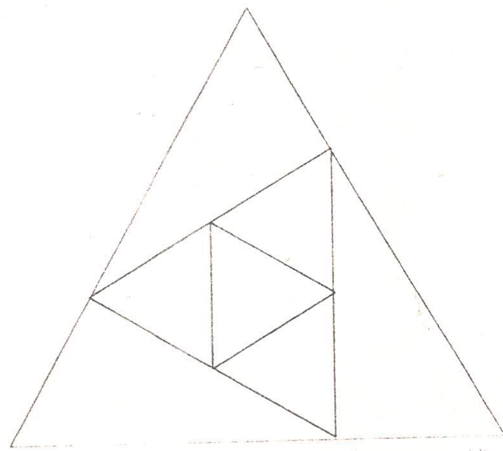
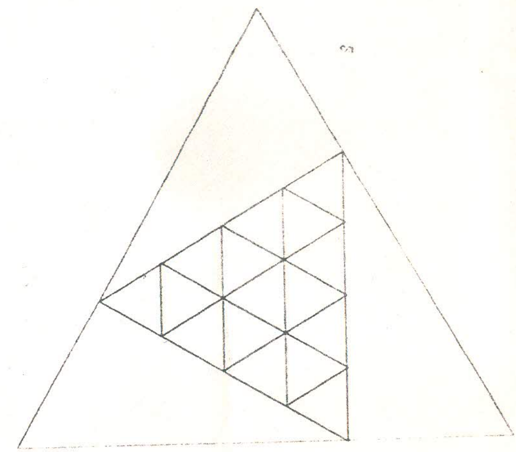


30 RODS

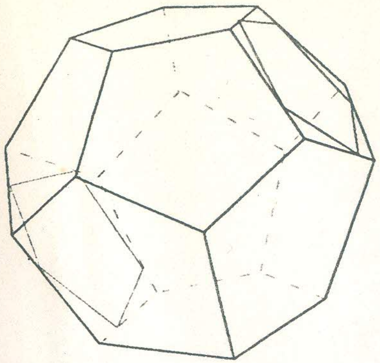


60 RODS

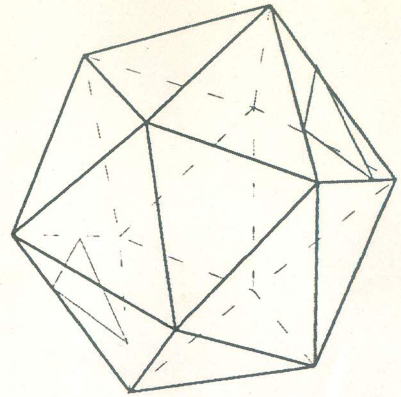


150 RODS

FIVEFOLD SYMMETRY IN 3D



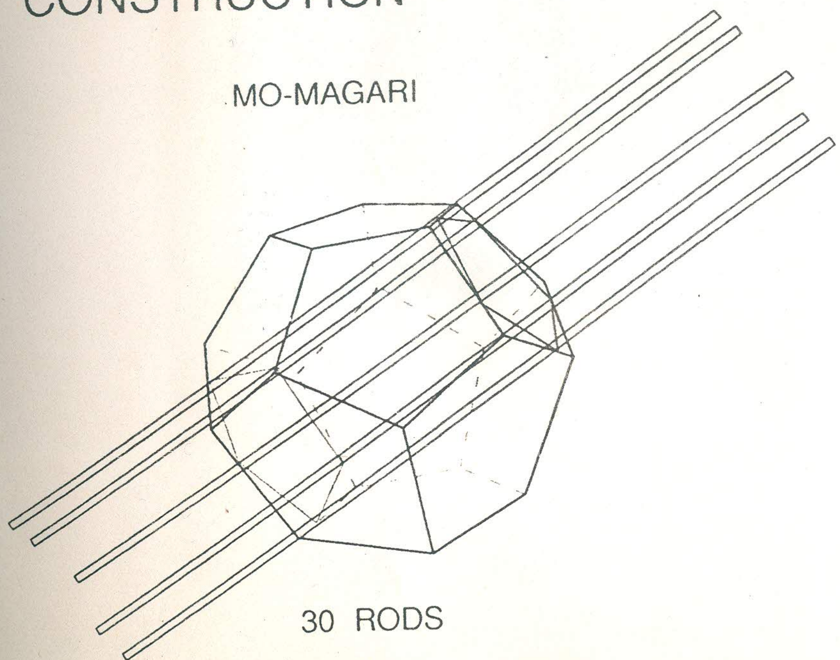
DODECAHEDRON



ISOSAHEDRON

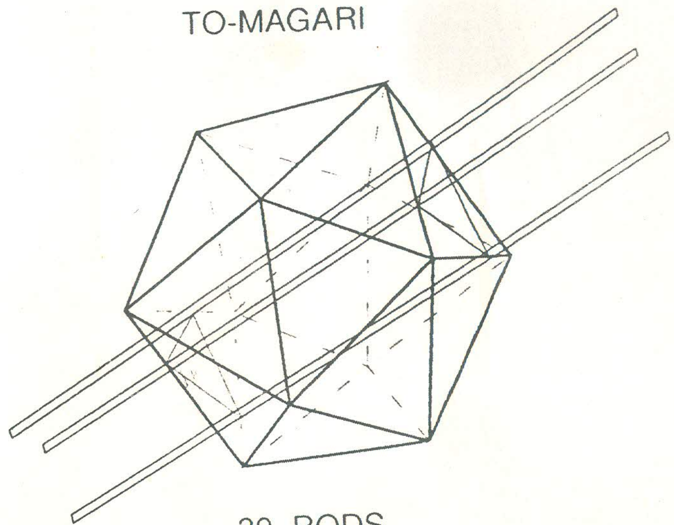
CONSTRUCTION

MO-MAGARI

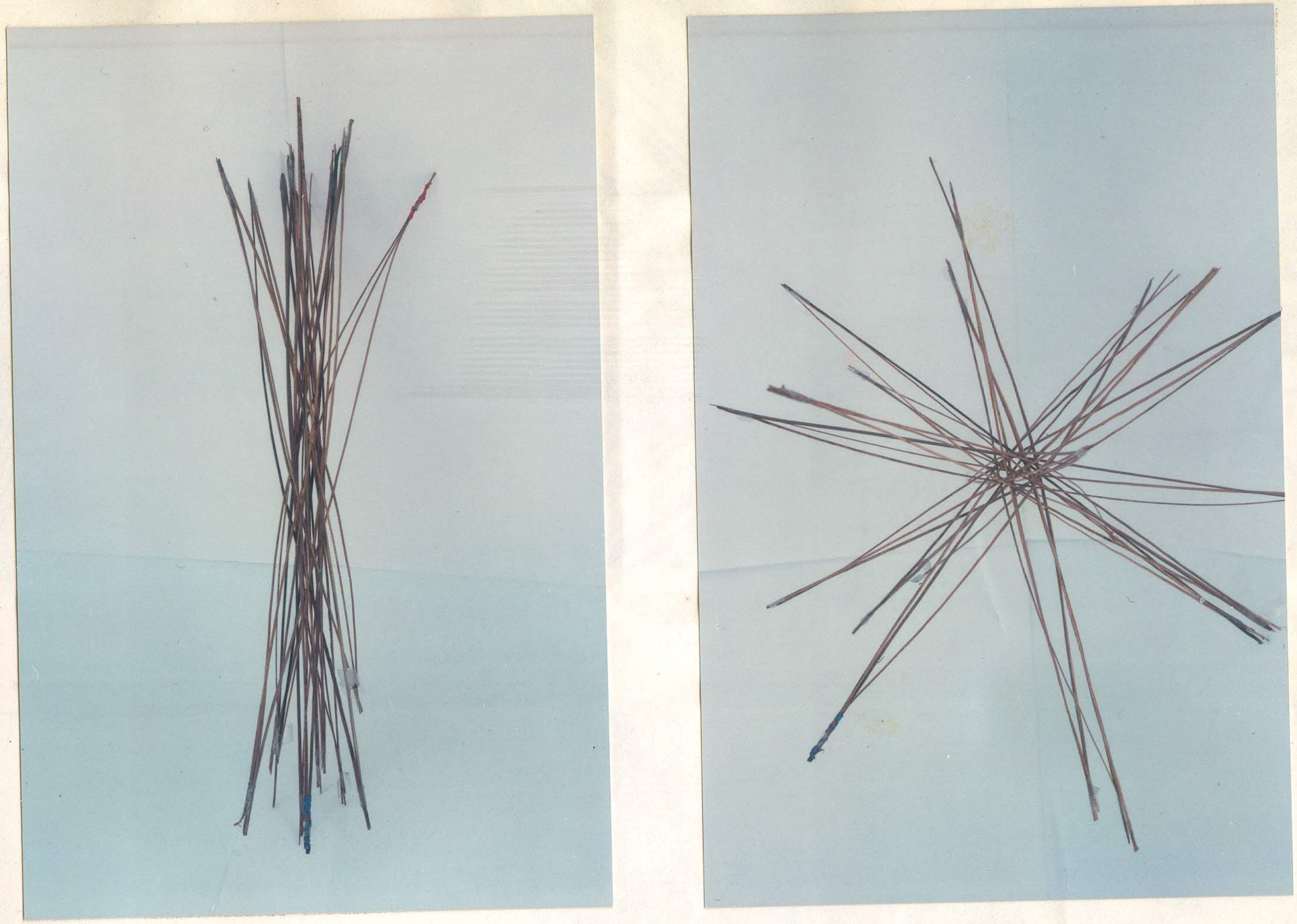


30 RODS

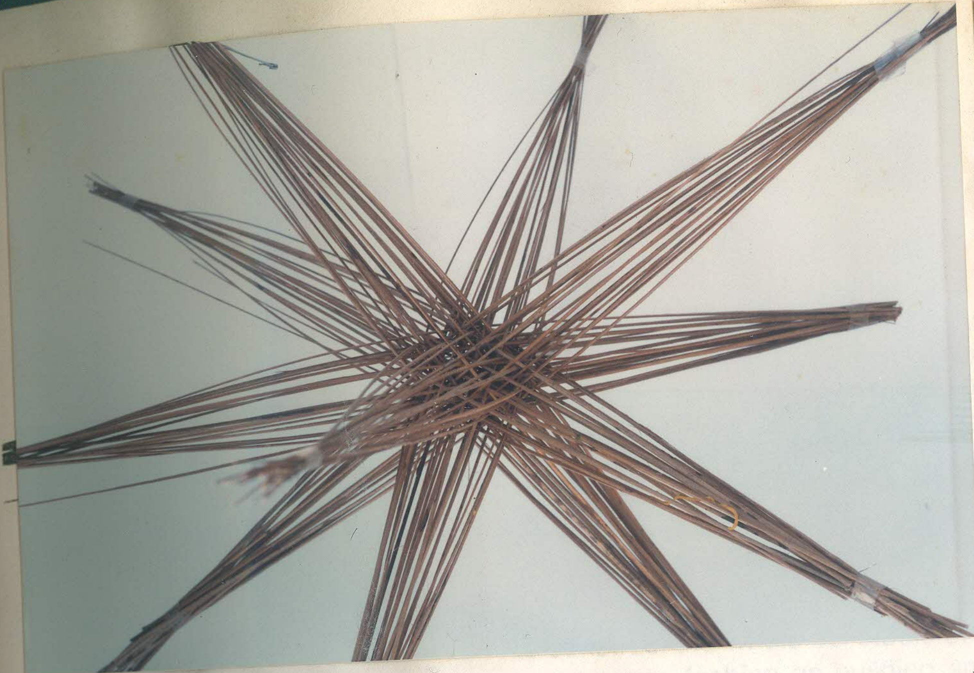
TO-MAGARI

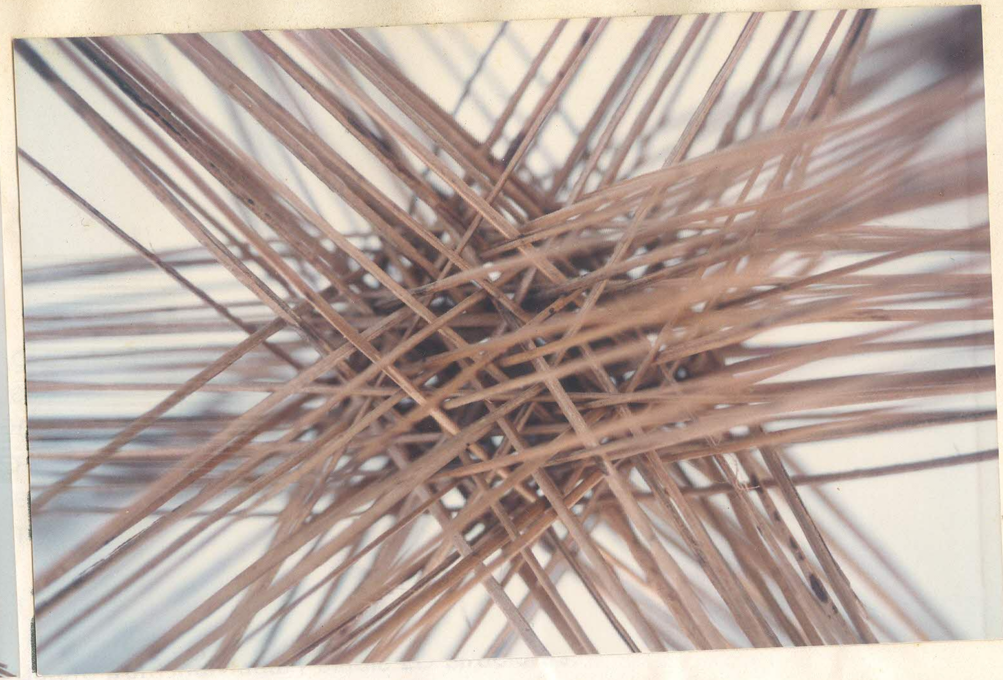


30 RODS



FOLDED AND UNFOLDED TO-MAGARI

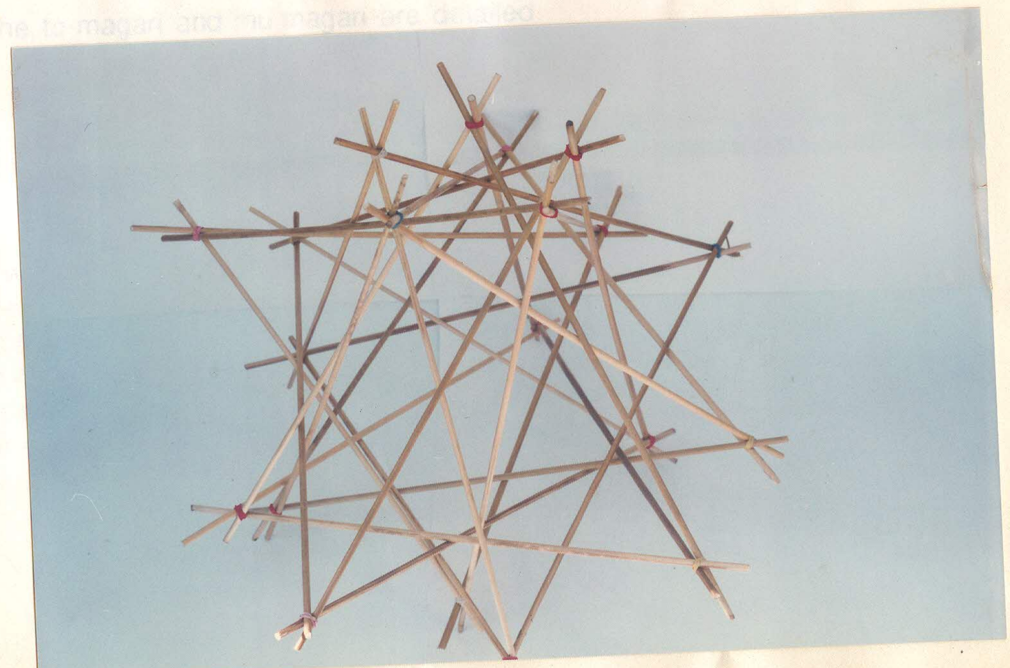




CORE OF 120 STICK TO-MAGARI

120 STICK TO-MAGARI

5 INTERTWINED TETRAHERONS TO FORM A DODECAHEDRON -MADE BY JOINING 3 COLSE STICKS IN A MU-MUGARI



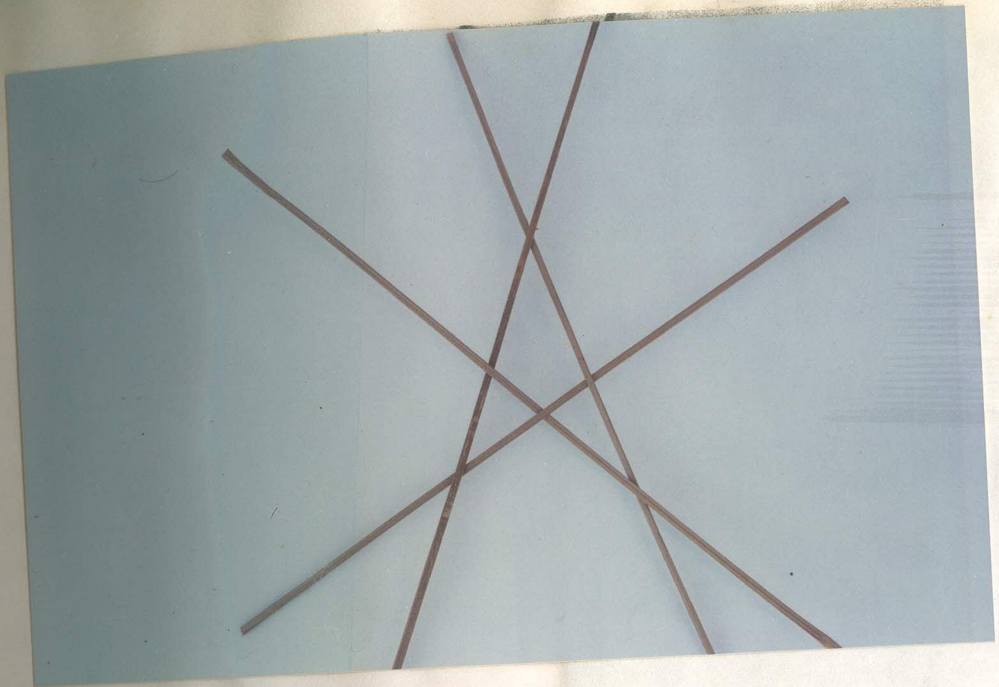
An alternative of the star cage was also made which was foldable like an umbrella using the TO-MAGARI method of construction. Other explorations included finding the connections between the to-magari and the mu-magari, in this direction it was found that in the to-magari if the parallel sticks were bound at their end and then connected to the closet set of similar points then the edges would give a dodecahedron and similarly in the case of a mu-magari we would get an icosahedron.

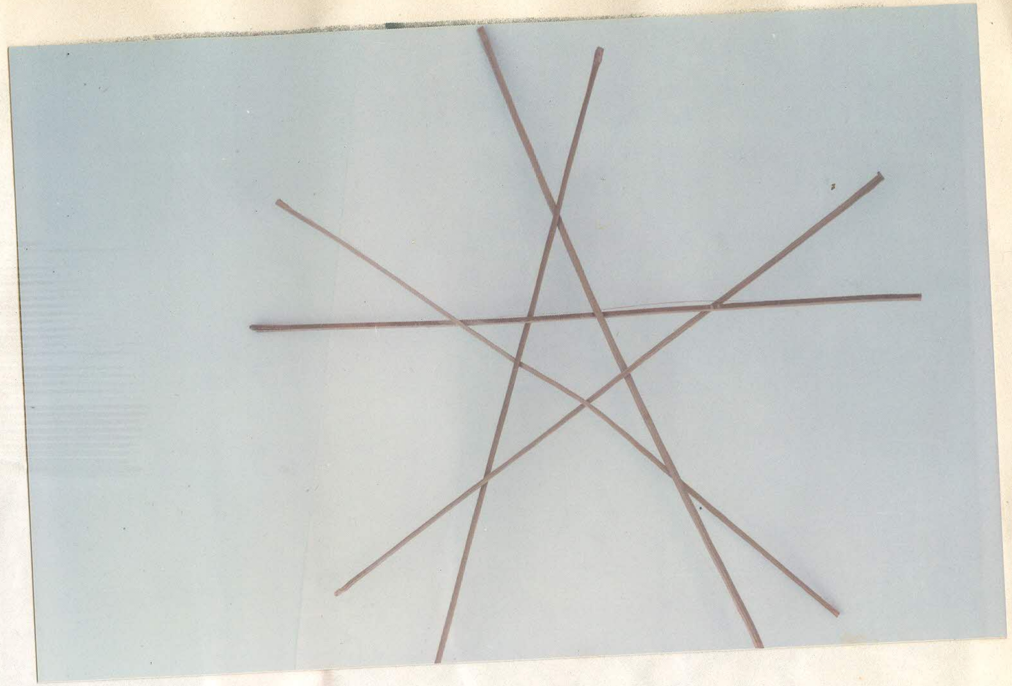
During the course of finding out the method of construction of the star cages I discovered a new method of constructing a 3D stable structure. That was to construct the whole 3D stable structure from individually stable 2D structures, in the same fashion as building any polyhedron using corresponding polygons on the sides.

The method for the construction of the to-magari and mu-magari are detailed here.

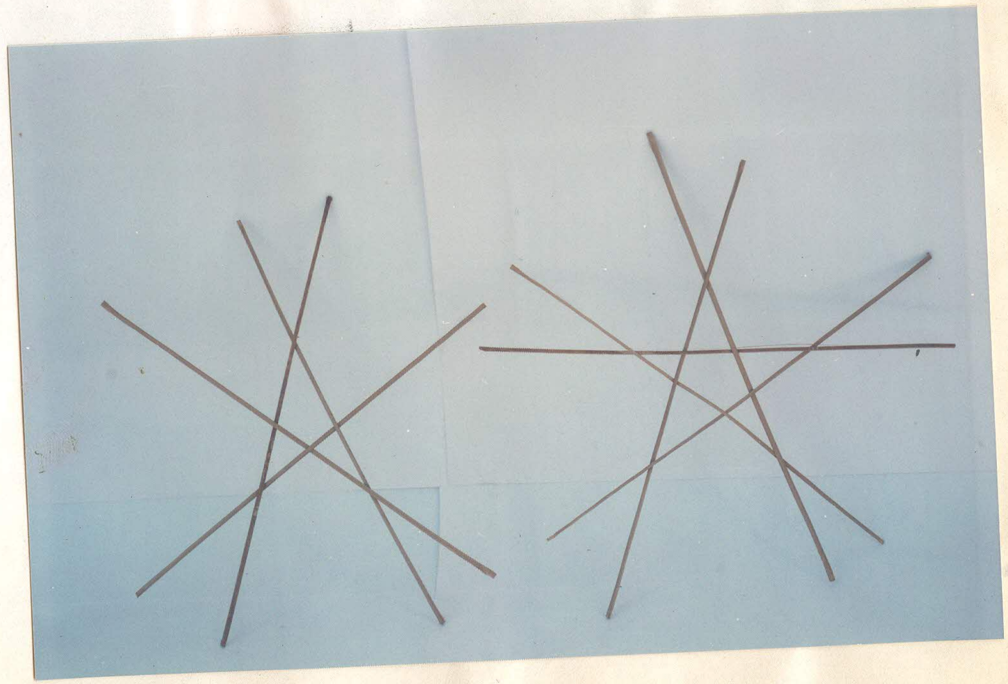
EXPLORATION OF 2D STABLE WEAVES

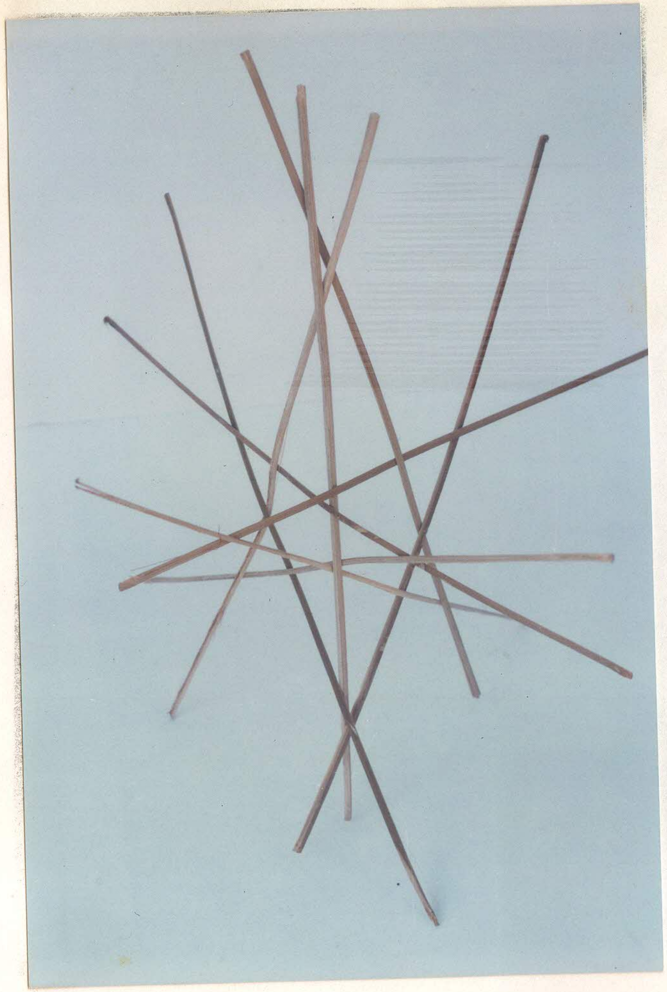
The exploration commenced with an effort to find the basic and most elemental 2D stable structure. Weaves using 2,3,4,5,6,.. sticks were used to get a stable structure. It is noted that it is not possible to make a stable 2D weaves with less than 3 sticks, also with 3 and 4 stick it was necessary to get its multiple, namely 6 and 8 sticks to make them stable. It was surprising to note that the elemental structure was in fact a form of the pentagram with four sticks instead of five but in the same formation. It was noted that the 5 stick combination was more stable than the 4 stick structure. All combinations with more than 5 sticks were found to be inherently stable.



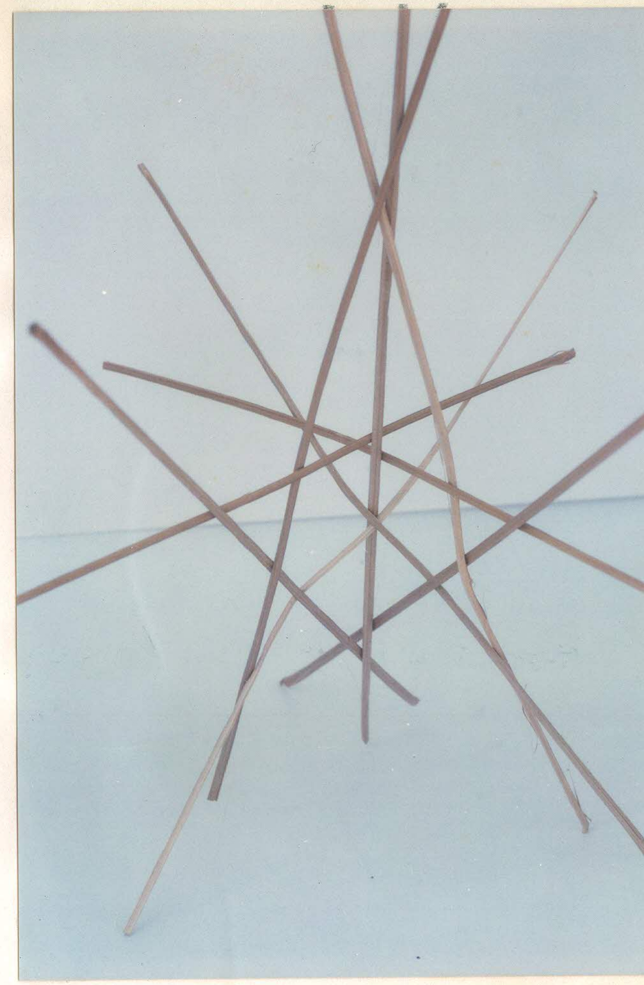


2D STABLE WEAVES USING 4 AND 5 STICKS

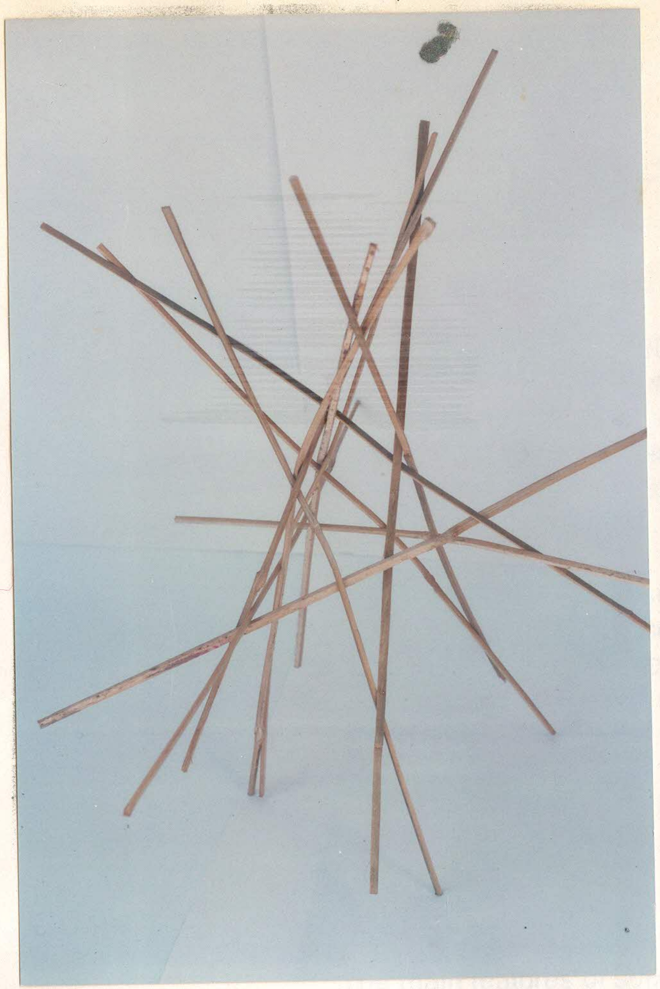




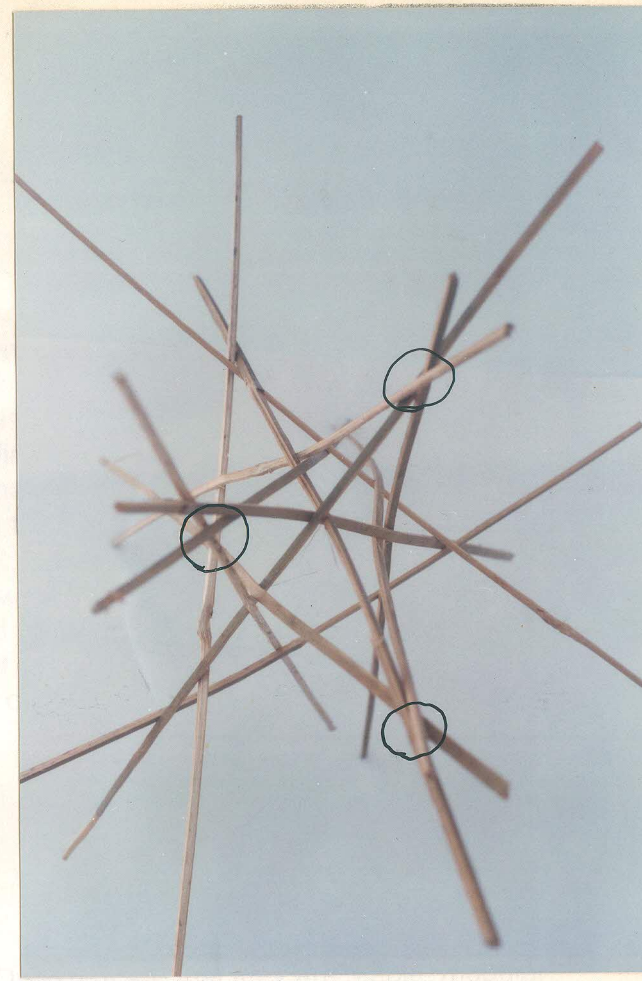
3D STABLE STRUCTURE
USING 4 STICK 2D WEAVE



2 FACE BUILDUP USING
5 STICK 2D WEAVE



3D COLLAPSIBLE STRUCTURE
USING 5 STICK 2D WEAVE



TOP VIEW OF 3D STRUCTURE
CIRCLES INDICATE POINTS
THROUGH WHICH TENSION
MEMBERS RUNS.

EXPLORATION OF 3D STRUCTURES USING 2D STABLE WEAVES

Once the basic stable 2D weave was made, the effort was directed towards making the most elemental 3D structure, which is tertrahedral using the pentagonal 2D weaves. It was also discovered in this process that there would be two ways of starting the weave, first would be to go about connecting the edges of the 2D weaves in a ring formation and second to connect the edges of the 2D weaves to form corners. Both these methods were used to make basic structures.

The main feature of these 3D weaves are that they are more stable than the star cages. In case of the second case where the elements are connected together to form a corner we see that the whole structure is collapsible, hence such a structure can be bought up or down with the help of tension members.

POSSIBILITIES

The main features of such a 3D system are that they are stable, modular, has high sculptural value, can be used in other field of science as models for study etc.

The possible uses of the 3D stable structures could be in such areas as

1- 3D sculptures
2- Architecture
3- Display systems
4- Modular and dismantleable systems
5- Basket making

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4 - STAR CAGE - NEW DIMENSION OF THE PENROSE LATTICE
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