B.Des Project 1

Designing an Algebra Training Tool for Secondary School Students

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Project Approval

This is to certify that the project entitled “Design of an Algebra Training Tool for Secondary School Students” by Vineet Srivastava (16U130002) is approved in partial fulfilment of the requirements for the Bachelor of Design degree.

Project Guide: ____________________

Chairperson: ____________________

Internal Examiner: ____________________

External Examiner: ____________________

Declaration

The undersigned has conducted the work for the project titled “Design of an Algebra Training Tool for Secondary School Students”, with adherence to academic honesty and integrity, as a part of the undergraduate project under the guidance and supervision of Prof. Swati Pal.

The undersigned hereby declares that this is his original work and has not been plagiarized in part or full form. Furthermore, all sources of material - visuals, designs etc. have been properly cited and acknowledged.

I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action by parties whose content was used for this project without proper citation or permission.

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Abstract

The COVID-19 pandemic of 2020, and the policies of social distancing and lockdowns that followed it, led to an unprecedented shutdown of public activity across the world. Students and educational institutions were placed under sudden stress, with the World Bank estimating that at least 1.5 billion students and 63 million educators faced a sudden disruption in their academic practices.

The general response of educational institutions was to expand their digital infrastructure to enable online or e-learning, but this capacity varies greatly between institutions, owing to differences in access to resources and infrastructure.

In this project, I study the impacts of the switch to e-learning on secondary school students studying mathematics. I chose to target the problem of assessment as a means of improving overall learning outcomes in the e-learning environment. While blending learning, practice, and assessment is not a new idea in e-learning, existing tools of this kind offer limited modes of interaction, most often just a number as a final answer to type into an input box.

To address this gap of interactivity—which limits an understanding of the process the student uses to solve problems, which further leads to a gap in what the tool can measure—I draw on existing research in mathematics pedagogy and e-learning, as well as direct interaction with secondary school students and teachers, to produce a concept for a tool that trains students of Class 9 and 10 in algebra. The tool offers a variety of novel modes of interaction with algebra problems, a progressive difficulty increase that allows students to progress from a clear understanding of concepts to accurately executing multi-step problems, the ability to track their performance in different topics and basic arithmetic skills, and sharing that information with their teachers.

This approach to a practice tool enables the isolation of basic conceptual abilities from those relating to arithmetic and procedural fluency, enabling better insights from the objective assessment format. These insights can be used by teachers to better plan their class hours and address difficulties that may have been unrecognised otherwise.

The concept can be extended to higher secondary topics such as coordinate geometry and conic sections. It is also possible to integrate a system like this with existing graphing and geometry tools for a much more tangible e-learning experience of secondary school mathematics. The information structure of the tool also allows in principle for the addition of community questions and answers, while maintaining the progressive difficulty increases.

Introduction

An Brief History of Online Learning

Online learning, or e-learning, is a broad term that refers to several means of using technology to assist in some kind of learning activity. Content, in the form of text, audio or visuals, is delivered to the user in different relational environments (Brooks & Bichsel, 2014). The first attempt at this in the modern era is thought to be the PLATO (Programmed Logic for Automatic Teaching Operations), developed by the University of Illinois in 1962. It could be programmed with customisable sequences and hardware to perform a range of functions (Bitzer & Braunfeld, 1962).

With the advent of the internet at the turn of the millennium and the proliferation of technology-based service companies that followed, a range of different free and paid e-learning tools exist online today that provide technology-assisted learning through many different modes, such as computer-managed learning (CML), computer-assisted learning (CAI), and synchronous online learning, to name a few. These modes refer to the direction of information flow, interactivity, and temporality of the relationship between student, teacher and technology in each case.

Blended learning is a term that refers to a learning mode that combines elements of different modes, most commonly blending aspects of traditional classrooms with e-learning tools, in an attempt to optimise the benefits of each.

Online Learning in 2020

The COVID-19 pandemic wrought unprecedented disruption upon the world. With over 1.5 million registered deaths globally as of 1st December, 2020, it is the most deadly pandemic since the Spanish Flu of 1918. Countries dealt with the disease with a mixture of enforced lockdowns and social distancing norms, both of which required temporary closures of spaces where large numbers of people might ordinarily gather, due to the risk they presented. While essential services such as hospitals, groceries and banks largely remained operational, public transport, shopping areas, movie theatres, and—perhaps most critically—schools and universities were forced to close campuses.

In absence of campus facilities, e-learning was an obvious fallback for educational institutions. However, a sudden switch to e-learning requires certain prerequisites: information about and access to e-learning tools, hardware, internet infrastructure, and technical literacy among teachers and students. These factors vary greatly across the world, with the World Bank (2020) reporting that India still only has around 50% internet penetration. This gap in access, and the resulting difference in the severity of impact of the COVID-19 school closures on people based on their access to digital services has been described in contemporary scholarship as the “digital divide” (Ramsetty, 2020).

The Indian Context

It is in fact very difficult to provide a simple description of “the Indian context” of the digital divide, as the picture varies very much based on what kinds of schools, in what regions one might choose to look at. Well-funded schools in urban regions with high-speed internet connectivity, tech-literate students and teachers, as well as access to smartphones and computers have a range of options available to them when they formulate their e-learning strategies. Conversely, public schools with a shortage of teachers and a lack of internet access or digital hardware among students do not have a realistic shot at functioning in the e-learning paradigm, as discussing an e-learning strategy in an environment with no computers or internet is like designing a cruise control for a car with no wheels.

Consequently, this project will have to focus on a more limited slice of India with more similar issues in the context of education in the COVID-19 era.

Chosen Context

This project functions in the limited context of online mathematics learning for secondary school students and teachers from several schools in Vellore, Tamil Nadu. That being said, several aspects of the chosen context are likely common to other tier-2 and tier-3 cities in India, and even with rural India as it continues to industrialise into the future.

Relevance

India is vulnerable to the digital divide, owing to its modest penetration of both digital and educational infrastructure. While the needs of a large section of the population cannot currently be addressed via e-learning—at least in its generally understood sense—there are still a very large number of students and teachers who are new adopters of the paradigm, born out of a crisis. As we will discuss, even those with access to e-learning face difficulties of various kinds.

Primary Research

Primary research for this project consisted of several kinds of interactions with mathematics teachers and students from local schools. These consisted of:

1. Online survey of teachers
2. Telephonic interviews of teachers
3. Passive observation of online classes
4. Telephonic discussions with students

Online Survey of Teachers

The first data collection activity I conducted was an online survey of teachers. The survey questions covered the teachers' experience and comfort with technology in general and online teaching specifically, the different modes of e-learning they used, perceived advantages and drawbacks of the switch to online teaching, perceived impact of the switch to online teaching on their teaching process and learning outcomes, and the effect of the switch to online learning on students' performance, attitude, and relationship with the teacher and class.

The survey received 23 responses from teachers of mostly Class 9 and Class 10. The main findings from the survey were:

1. The majority of teachers are not especially comfortable with computers and new technology
2. e-Learning for the teachers surveyed primarily meant live lectures on real-time-communication (RTC) platforms such as Zoom, digital submission of assignments, and objective online quizzes on platforms such as Google Forms
3. Most teachers felt that the switch to online learning had a net negative impact on their students' learning and engagement
4. Almost every teacher highlighted the reduction in student feedback as an impediment to their teaching process
5. According to the teachers, the main advantages of e-learning come from the higher self-engagement required from students, and to a lesser extent, the wide range of available external tools and resources
6. Teachers identified more drawbacks with more frequency, such as difficulty in tracking individual students and personalising teaching approaches, diminished teacher-student interaction, and a lack of practical learning opportunities
7. The teachers did not share any common assessment of what topics or ideas were harder to address online: they each had their own ideas

Telephonic Interviews of Teachers

After collecting the contact information of the survey respondents that opted to share it, I conducted telephonic interviews with them to discuss the themes above in more detail, as well as their experiences and teaching processes before the pandemic. I conducted 11 interviews in total.

The single biggest issue the teachers raised was the removal of traditional feedback channels in the online format: all of their schools were conducting live lectures via RTC platforms, so while the students could see the teachers work through problems in a notebook or on a whiteboard, the teachers' view of their class was of black screens and muted microphones. Additionally, most teachers did not have a laptop or desktop PC, and complained that the RTC platforms (MSTeams in particular) were not user-friendly on their smartphones, and did not let them view their students. The lack of feedback means that teachers are unable to check if their students are learning, following along, or even just listening. As in the survey, the teachers suggested that online learning works well for high-performing students who respond well to self-directed learning, but that online learning also makes it easy for struggling students to completely slip under the radar. Most teachers expected an overall drop in academic performance this year.

Passive Observation of Online Classes

Some of the teachers I interviewed agreed to let me sit in on their online classes, and I made some observations in the process.

The classes are attended by around 30 students, although the number doubles occasionally when another class' teacher is unavailable and two classes are combined. Although there was some individual variation, the general structure of the class consisted of going through problems while discussing the concepts and processes associated with them. The video feed consisted of some kind of canvas, and the teacher would write on it as they discussed problems.

In this discussion, most teachers made an effort to be interactive and request contribution/participation from the students, but only a small fraction of students would respond (around 10%) via voice or chat when questions were called out by the teachers.

This format of online class is not very visually engaging for the student, especially by modern standards for digital video content. The format relies heavily on the teacher's capacity to engage students, and the option of live interaction with the teacher is what maintains the value of the format over a more polished, pre-made lecture. However, the value of interaction is admittedly diminished when only around 10% of the class makes use of that facility.

Teachers in the given context are also limited in terms of ability to significantly changing the format of their live lectures, since many of them are:

1. Not especially comfortable with technology, making some kinds of content creation inaccessible
2. Limited to using smartphones, making it harder in practice to share screens and link other forms of content

Telephonic Discussions with Students

I conducted telephonic interviews with 7 students on their attitudes on mathematics, as well as their performance and confidence in the subject. I also asked them what they thought of the switch to online classes.

I found that relatively few students said they or their friends liked mathematics, with the majority being indifferent or negatively disposed. A dislike of the subject seemed to negatively impact performance, and especially confidence.

As far as particular mathematical topics were concerned, geometry and mensuration was the most popular topic, even being described as “fun”, or otherwise “easy”, “simple”.

Algebra was particularly unpopular, especially amongst students who disliked mathematics, citing its perceived irrelevance to their daily lives and its abstract nature. Interestingly, a majority of these students said they liked physics/science (as applicable by board).

The switch to online classes was seen largely unfavourably by the students interviewed, who largely wanted to be able to go back to school to meet their friends. They also confirmed that they were more than willing to leave their classes connected in the background while they did something else. Nearly all the students said they found it harder to pay attention to classes online, with only one saying they found no difference.

Summary

Teachers need to understand their students' learning progression to design appropriate teaching strategies. Pre-pandemic, they could observe students, ask them questions and review their work. The current format of online classes eliminates all these feedback channels. The only class-wide feedback that remains are the assessments they conduct: either digitally submitted worksheets (solved on paper) or objective quizzes on platforms like Google Forms. However, conducting more assessment can quickly become labour-intensive for both students and teachers.

Online classes are also challenging for students. The format is not very visually engaging, and they find it harder to pay attention when not physically in class. High performers respond well to self-direction, but struggling students are left behind. The problem is compounded by the relative antipathy of students towards mathematics. However, it must be noted that students were much more willing to accept and enjoy mathematics packaged in a more relatable or relevant form, such as geometry or physics. This suggests that their attitude to the subject could be flexible, based on the exposure they receive.

Insights

Combining research from students and teachers, we find:

1. Online learning disadvantages weaker students in its current form, due to the self-direction requirement
2. Assessments are the only current feedback channel, but they quickly become labour-intensive
3. Lack of feedback makes it easier for weaker students to slip under the teacher's radar
4. Students either struggle with mathematics due to weak concepts or poor numerical skills
5. Students have a lower attention span online

Contextual Factors

The research conducted set some context for the project, which any solution must account for:

1. Teachers are not trained to use online teaching methods
2. Schools may not provide teachers supporting infrastructure, such as home broadband
3. Schools are otherwise equipped with staff and infrastructure: some classes shot in school
4. Students are generally more familiar with technology than their teachers

Design Opportunities

Based on the research conducted with the teachers, I identified four major design opportunities.

Enabling Greater Interaction During Lessons

This would involve exploring new modes of dynamic interaction that could work during live lectures. Solutions would be interactive tools that enable teachers to better interact with their classes as a group as well as individually.

This focus area was rejected due to heavy technical requirements. Concepts could be generated for new means of student-teacher interaction during lessons, such as shared whiteboards, passing a problem around for students to each solve a step of, or creating daily/weekly class groups to perform activities. However, the implementation of tools for real-time communication is challenging, and additionally would not fit the problem context. Such a tool would require significant maintenance that is not feasible for the kinds of schools dealt with in this project.

Creation of Teaching Aids for Concepts

This would involve creating physical or digital widgets that help illustrate mathematical concepts for students. Solutions would be tools the teacher could integrate or share with students.

This was rejected due for two reasons.

1. Market saturation: there are entire websites devoted to this sort of applet, making it difficult to create a differentiated product.
2. Despite a glut of such tools, the teachers have made no move to adopt them. In fact, the teachers showed little interest in using teaching aids in general.

Comparison of Methods for Content Delivery

This would involve a comparative study of different means of content delivery: video, text, infographic, interactive applets, to evaluate their efficacy at conveying particular concepts. The study would make use of existing content.

This was rejected due to the realistic sample sizes I would have been able to achieve, and the limited value such analysis would therefore provide. Additionally, the intent of the project was to identify and design an intervention that would help improve learning outcomes in the e-learning era, and this does not address that.

Streamlining Continuous Assessment

This would involve exploring new modes of assessment that provide the teacher with richer feedback about their students, while efficiently managing labour required to complete and evaluate the assessments.

This was selected as the area of focus, as the infrastructure and technical requirements of these solutions do not necessarily have to be very great. Additionally, continuous assessment can be used in conjunction with class hours for more effective management of student’s study hours and attention span. This will be discussed in greater detail later.

Secondary Research

With a problem (lack of feedback channels to teachers about student learning when using e-learning), a context (the schools studied in Vellore), and a design opportunity identified (continuous assessment as a feedback channel), the next step was to conduct a more in-depth review of existing scholarship that is pertinent to the project.

Rationale

The biggest problem identified during primary research was the lack of interaction between students and teachers. Teachers are left in the dark about what their students are doing, while students find it difficult to engage with a one-way screen for long stretches of time. This lack of information transfer is a hindrance to the efficacy of the teaching-learning problem.

The most direct way to address this issue is to open new student-teacher communication modes during the live lectures, but this approach has been discarded for the reasons discussed in the previous section.

As of now, assessments are the de facto feedback channel for teachers. They are primarily conducted in two forms.

1. Worksheets: these are sheets of problems which students are expected to work through on paper and send the teacher a scanned copy of their work. A student worksheet can tell the teacher a lot, but worksheets are a lot of work for a teacher to create and correct, as well as for students to work through. They are also compulsory homework, and therefore do not take students' personal learning requirements into account.
2. Quizzes: these are objective quizzes conducted on Google Forms or similar platforms. These are very easy to evaluate and fairly easy to set. However, the drawback is that each question tells the teacher very little about the student's learning. Objective tests can quantify success, but they do not easily lend themselves to diagnosing reasons for failure i.e. the teacher cannot make a clear assessment of why a student has gotten a particular question wrong.

Keeping this in mind, a new assessment tool that manages to combine the advantages of these two assessment types would provide richer feedback to the teacher without making their labour requirement, or that of their students, unmanageable.

There are, therefore, three key areas of study required to develop such a tool:

1. Mathematics Learning Models
2. Mathematics Assessment
3. Existing Market Tools

Mathematics Learning Models

Knisley (2002) described a four-phase mathematics learning model based on Kolb's learning style inventory (1984). It describes four phases of learning of mathematical ideas, namely allegorisation, integration, analysis and synthesis. The four phases describe how new concepts are first integrated in terms of existing knowledge, are assimilated into the learner's knowledge base, and may then be used to acquire further knowledge. Knisley also describes a change in approach to be used as the learner progresses through the stages, with teaching approaches going from concrete to abstract with learner achievement.

Wittrock (1974) described a shift from behaviourism to cognitivism in understanding mathematics learning. Behaviourism here refers to an outcome-based approach (understanding learning by studying *behaviour*) as opposed to a cognitivist approach, which seeks to understand the basic elements that lead to the observable behaviour. Wittrock also describes the relationship between new and existing knowledge, and suggests a similar transition from reception teaching (in which the student receives knowledge) to discovery teaching (in which the student is stimulated by the teacher to connect new with existing knowledge).

This argument is similar to one made by Vygotsky (1978) that describes a “zone of proximal development” which refers to new knowledge a person can acquire, that is close enough to the person’s existing knowledge space.

Sukardjo & Salam (2020) investigated the relationship between students’ self-directed learning (SDL) scores and their response to concept attainment model (CAM) learning as opposed to direct learning. SDL is a measure of student’s learning autonomy and proactivity (Knowles, 1975),CAM is a more open ended model of learning based on classifying *exemplars* as belonging to, or not belonging to the concept (Joyce & Weil, 1997). They found that while high SDL students benefited from CAM, low SDL students benefitted more from direct learning.

Some additional information:

1. Collaborative groups of students of various performance abilities known as student-led achievement divisions(STADs) have been shown to improve learning outcomes
2. Realistic and culturally relevant examples improve learning outcomes (Widada et al., 2018)
3. Semi-concrete media such as flash cards and illustrations improve mathematical performance even amongst high school students (Widodo & Wahyudin, 2018)
4. Factors and measurable indices such as counting ability, working memory and awareness of mathematical pattern and structure (AMPS) strongly correlate with mathematical performance (Passolunghi et al., 2007; Mulligan, 2011)

Mathematics Assessment

Kulm (1994) drew a distinction between procedural and conceptual assessment in mathematics. Broadly, conceptual assessment tests whether a student understands a concept i.e. a mathematical structure/relationship and how that structure behaves. Procedural assessment, on the other hand, measures the student’s ability to utilise the concept to accurately execute a procedure by which to solve the problem. For example, understanding what division means compared to dividing two polynomials. Concepts and procedures are always layered to an extent, but it is possible to distinguish between them.

Torrance (2007) commented on the shifting relationship between assessment and learning, from assessment of learning, to assessment for learning, to assessment as learning. He argues that the growing prevalence and importance of standardised assessment has led to a convergence of assessment and learning.

Shorter & Young (2011) showed that continuous assessment is a better predictor of learning outcomes than either projects or traditional homework.

Shapiro (2011) classified a group of students into three performance levels (frustration, instructional, mastery) which are themselves based off of Vygotsky's zone of proximal development. The classification was done based on a test of calculative speed and accuracy. Shapiro showed that the frustrational students benefited more from directly copying problems, while the others benefited from being timed on their questions.

Low & Hew (2020) performed a comparative study comparing flipped classrooms, traditional classrooms, and independent online study. The flipped classrooms produced the best performance, followed by the traditional classrooms. However, Summers et al. (2005) showed that students may report lower satisfaction from online courses, despite essentially equal performance. Edwards et al. (2013) has also shown equivalent outcomes in children learning online.

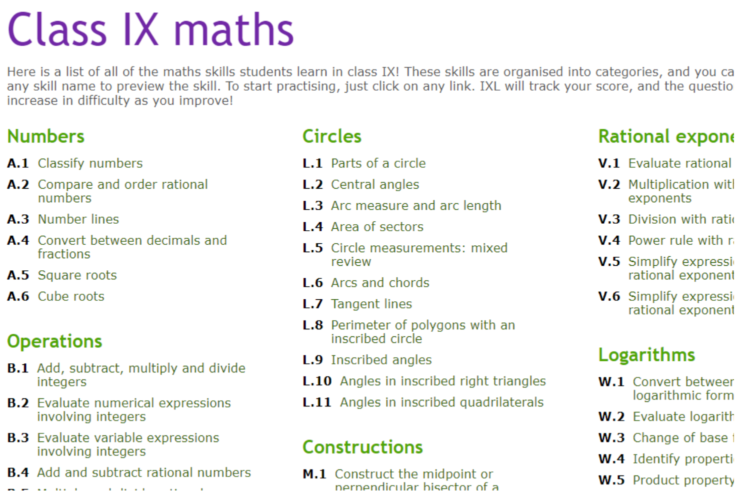
A number of researchers have also measured the impact of computer-generated homework/practice on students and found positive results. These are tools that procedurally generate any number of practice problems to work on (Kodippili & Senaratne, 2008; Edwards et al., 2013; Emerson & Mencken, 2011).

Market Analysis

I studied 12 popular online mathematics platforms, to understand their features and purpose.

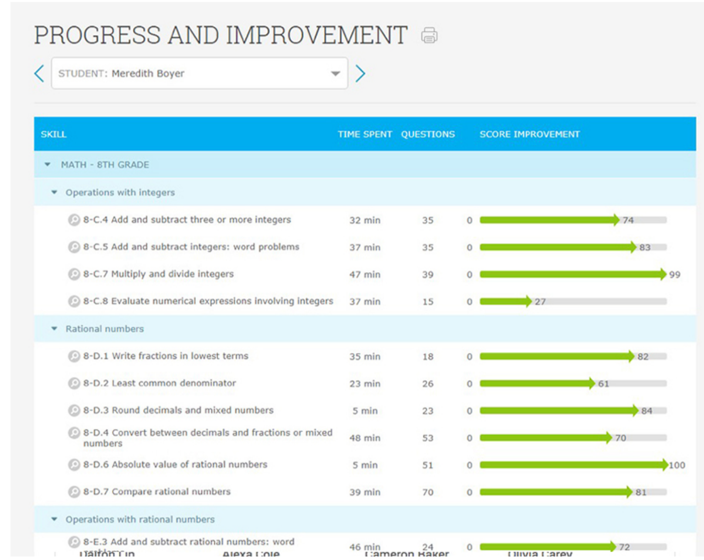
They have the following main features.

1. Skill Classification: this is a list of mathematical skills (often based on an educational curriculum) that a student may practice or work on. Skill classifications enable the tool to be somewhat systematic about helping students practice mathematics. IXL's skill classification page is shown below.



One of *IXL's skill classification pages*

2. Student Profile: the mapping of a skill classification onto a student registered with the platform makes a student profile. It is a unique object that every user has, with their achievement in each available skill recorded. Student profiles and skill classifications usually go together, since the utility of each is greatly dependent on the other. A sample of IXL's student profile is shown in the next column.

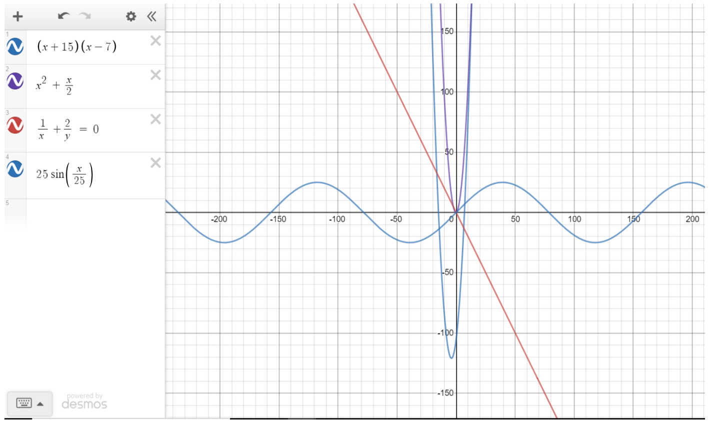


A sample of IXL's student profile page

3. Collaboration: this is implemented in different ways, but discusses whether the platform is community-based. Some platforms are entirely between the user and the server (website), while in other platforms, the server provides a base of operations upon which a community adds its own problems and solutions to expand the platform.

4. Illustrative Math: this refers to different features designed to visualise mathematics and make it more tangible. These tools are most often used as teaching aids, and their interactive methods have not been transferred to practice and learning sites. Desmos graphing calculator is depicted below. It enables the graphing of nearly any mathematical function, and

helps students improve their ability to visualise mathematical expressions by making it easy to interact with them.



Desmos graphing calculator

5. Question Model: this refers to whether a platform has a hint, explanation, or model question that explains the basics of the problem to the reader. It is a common feature on practice and learning sites, as the extent of interaction present in most.

Summary

Several important conclusions can be drawn from the research reviewed. First, the notion of mathematical ability is something that exists, and for which sufficiently reliable, measurable proxies are available (Mulligan, 2011; Passolunghi et al., 2007).

Second, the teaching/learning approach must vary based on the skill of the user: from more explicit and direct modes of learning at low levels to abstract and stimulatory modes at higher performance (Sukardjo & Salam, 2020). In a similar vein, culturally relevant examples in problems, as well as semi-concrete media in the classroom, improve learner outcomes by concretising information (Walker et al., 2015; Widada et al., 2018).

Third, learning at each stage must be stimulatory. The progression from concrete to abstract representations of concepts must require students to generate missing insights as progress (Knisley, 2002; Wittrock, 1974).

Fourth, online classrooms can produce equivalent, if not superior results to regular classrooms when appropriately set up and designed (Lo & Hew, 2020; Summers et. al., 2005).

Finally, computer-generated practice tools can be highly effective at improving student performance. These tools can be designed to intelligently cycle through the course requirement to ensure the student gets the necessary practice in each topic (Kodippili & Senaratne, 2008; Emerson & Mencken, 2011).

Kulm’s (1994) distinction of procedural and conceptual assessment is highly applicable to the context of this project. This is because most existing tools reviewed do not allow for any procedural interaction and only accept a final answer to a stated problem. In absence of procedural interactions, it becomes difficult or even impossible to measure the student’s procedural ability. This is fine for general learning platforms, as they do not particularly care what methods their users use to solve problems. If anything, they encourage creativity in approach through community features.

However, it is a clear shortcoming for students and teachers in the given context, who are explicitly tested on certain problem-solving procedures in their school syllabus. Additionally, the primary research identified that many students struggle with mathematics because of a poor understanding of the topics they learnt in previous classes, and low confidence in the subject that leads to a greater likelihood of committing inadvertent errors (“careless mistakes”) when doing tests or exams.

From a study of existing mathematics learning and practice tools, we find two consistent gaps.

1. Existing tools are behavioural. This means they measure a final output. They can evaluate a skill deficiency, but not its reasons. The fact that existing tools have no means of assessing procedure is partly by design. Many platforms do not need or intend to make statements about users’ procedures for solving problems. Users are even encouraged to approach problems from unconventional angles. However, in the context of Indian secondary schools, where procedures for solving certain problems are an explicit part of the curriculum, a procedural tool adds value as well gives the teacher richer data about student performance.
2. Existing tools have limited interaction. Several geometry and graphing tools make use of interesting interactive elements, but these are closer to teaching aids than problem practice environments. Adding interactivity to a practice tool enables many more kinds of data capture that can be used to make.

All this considered, there is a gap in the market for a tool that addresses procedural skills and weaknesses, and combines enhanced interactivity with a problem practice tool.

Problem Reformulation

After conducting and synthesising primary and secondary research, a few key issues emerged:

1. Lack of student feedback to teachers hurts weaker students and reduces the efficacy of the classroom
2. Continuous assessment is identified as the most viable means of improving student feedback under current circumstances
3. An automated assessment tool allows for full scalability with efficient use of labour
4. The problem of designing an assessment tool involves balancing data richness/granularity with the scalability and labour time constraints of both students and teachers
5. Existing market tools cannot distinguish between conceptual and procedural errors (why the student makes mistakes)
6. A clear gap exists for a tool that trains and evaluates students' procedural ability as well

Reformulated Problem Statement

This leads to the revised problem statement:
"To design a continuous assessment tool that allows secondary school students of mathematics to practice their skills, identify recurring conceptual and procedural errors, and share that information with their teachers to enable more informed decisions about how to utilise contact hours."

Solution Requirements

Primary requirements:

1. The basic solution must require minimal teacher input (to address non tech-savvy teachers)
2. The solution must at least be more engaging than current practice/assessment methods
3. The solution must adapt to users' level of skill
4. The solution must address procedural skills and track concept dependencies
5. The solution must create and share student reports with teachers

Secondary requirements:

1. The solution should allow peer-to-peer collaboration
2. Teachers should be able to share a personalised topic list that students can choose to focus on
3. The solution should allow custom problem sets

Design Implications

1. The solution must be mobile-first or at least friendly
2. The solution must be able to generate dynamic problems based on set forms
3. The solution must enable procedural interaction, to enable procedural assessment
4. The form of problems may vary with student skill level
5. The solution could automate/abstract some operations for less skilled users to isolate assessment

Analysis

After completing research and synthesis, the project requirement is for an assessment tool that is differentiated from other practice/assessment tools on the market by its attempt to measure procedural skills by enabling and tracking procedural action.

A complete platform is not feasible within the scope of this project, so I will focus on a proof-of-concept: applying these principles to an unaddressed area to evaluate the concept.

The selected focus area for this project is CBSE algebra for Class 9 and 10. The reasons for selecting algebra are:

1. It forms the foundation of higher secondary mathematics, and student comfort with the fundamentals is critical
2. Primary research shows it is a challenging topic for students and often a barrier to understanding and enjoying mathematics
3. Current solutions are rudimentary in the context of this project's intervention on procedural assessment

Curricular Breakdown

I analysed the algebra curricula of the CBSE Class 9 and Class 10 syllabus. First, the pre-requisites for Class 9 were identified, under three main types.

1. Arithmetic prerequisites: skills related to manipulation of numbers, fractions, irrationals, and other arithmetic operations
2. Conceptual prerequisites: mathematical ideas the student is expected to know, such as what variables, constants, and coefficients are
3. Procedural prerequisites: techniques and procedures the student is expected to know, such as how to perform algebraic addition, multiplication

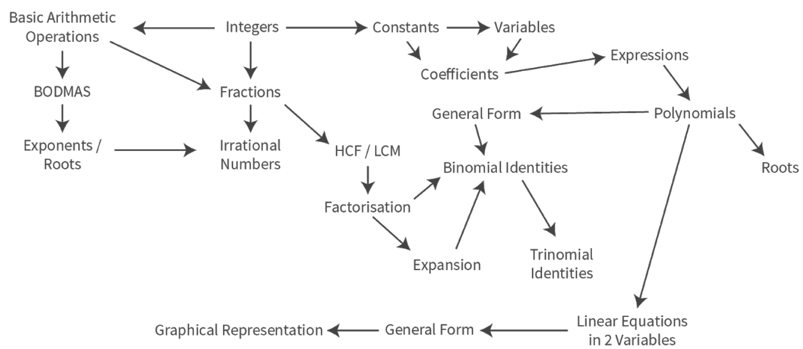
Each algebra chapter (topic) was broken down into subtopics, which were broken down into skills (procedural or conceptual units) with sub-skills (children of the parent skills) as appropriate. These skills were classified as either conceptual (green) or procedural (blue).

		Basics	Geometric Relationship with Linear Equations	
			Geometric Relationship of Pairs of Linear Equations	
New	Pair of Linear Equations in Two Variables	Graphical Method of Solution	Intersecting Lines	One Solution
				Algebraic Requirement
			Parallel Lines	No Solution
				Algebraic Requirement
			Coincident Lines	Infinite Solutions
				Algebraic Requirement
			Word Problems	
		Algebraic Method of Solution	Substitution Method	One Solution
				No Solution
				Infinite Solutions
			Word Problems	
			Elimination Method	One Solution
				No Solution
				Infinite Solutions
			Word Problems	
			Cross-Multiplication Method	One Solution
				No Solution
				Infinite Solutions
			Word Problems	
			Reducing Equations to Linear Form	
	Quadratic Equations	Basics	General Form	
		Solutions	Splitting the Middle	Concept
				Procedure
			Completing the Square	Concept
				Procedure
			Quadratic Formula	Procedure

Snapshot of Curricular Breakdown, Class 10

In most cases, each element in the curricular breakdown corresponds to a single question type. Some elements have multiple question types depending on how the question is framed, or the kinds of arguments (numbers or polynomials) those questions contain.

Curricular Dependencies



Curricular dependencies, Class 9

The next step after mapping the skills required in each class was to map their dependencies. They determine the order in which topics should be introduced to the student. From the diagram above, binomial identities cannot be introduced to a student that has not demonstrated an understanding of what factorisation is. Therefore, dependencies are a means of organising question sets within the curriculum based on what the student already knows. Further discussion of this will occur in the final concept section.

Notes

Most new concepts are introduced to students in Class 9, while Class 10 introduces far more procedures built on the concepts previously learnt. From the research, we saw that most student difficulties boiled down to two basic factors.

1. Weak arithmetic fluency: low confidence, procedural errors, and poor understanding of prerequisite topics
2. Weak concepts: poor understanding of polynomials, graphs, roots, and their relationship.

From this, we have three ability phases to look at for a user:

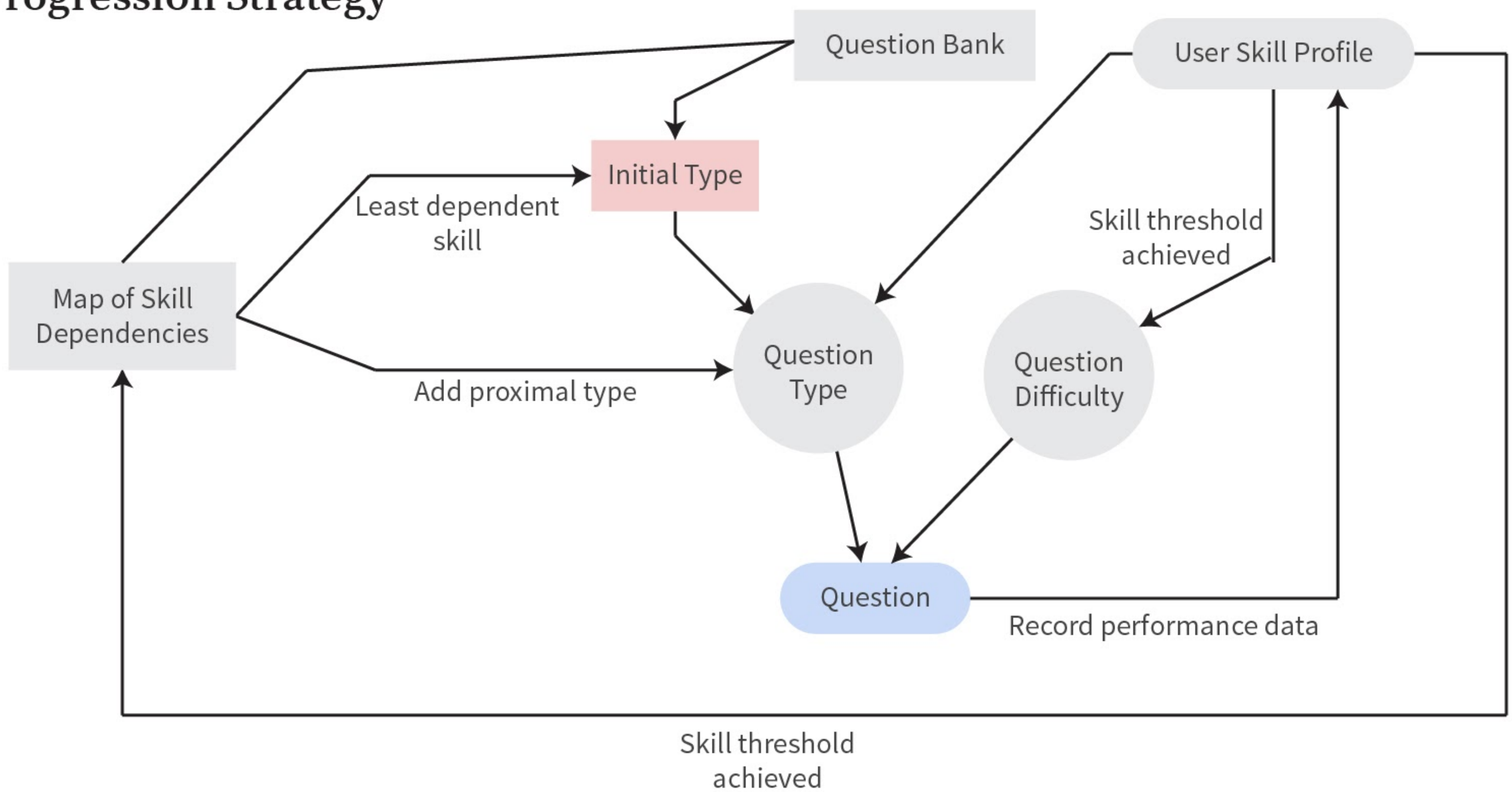
1. Foundational: introductory phase, minimum viable achievement of prerequisite skills
2. Conceptual: reliable understanding of the concept, procedural skill not yet fully developed
3. Procedural: accurate execution of problem-solving procedures

Progression Strategy

At this point, we have a list of all the kinds of questions in the syllabus, as well as the conceptual and procedural skills the user requires to solve them.

The progression strategy describes how the tool will respond to the user's performance to progress through the required topics. It makes use of Vygotsky's concept of the zone of proximal development, according to which the learner's capabilities can be enhanced with the help of guidance or "scaffolding". It is in this zone that learning is most effective. The user's performance is monitored, and the data is used to introduce new topics at an appropriate point, as well as change the difficulty of questions within a particular type in response to user performance data. The hierarchy is abstracted below:

Progression Strategy



The tool starts off with the question type that has the fewest dependencies (polynomial concept questions for Class 9). In response to user performance, new question types are introduced to the question set, and the difficulty of questions within types are modified as required. The skill dependency map introduces a skill in the dependency tree that is closest to the skill whose comfort threshold was crossed.

Concept Generation

At this stage, we have a full catalogue of skills/question types, their conceptual dependencies on each other, and a strategy for how to get the user to work through them. The next step of the project is to explore how the user is to interact with the practice questions.

To conduct procedural assessment, we need to be able to conduct procedural operations, which refer to any valid algebraic operations.

The standard algebraic operations are addition and subtraction,

$$a+b, \quad a-b$$

multiplication and division,

$$a \times b, \quad a \div b$$

Exponents and roots.

$$a^b, \quad \sqrt[b]{a}$$

There are generally two kinds of algebraic objects that the tool will have to deal with: equations and expressions. These operate by the same rules, but certain functions will work differently for the user based on whether they apply to an equation or an expression. This is an expression:

$$2x^3+8x^2-11x+7$$

Expressions are sequences of numbers, variables and operators that make a valid mathematical statement.

This is an equation:

$$y-7=4(x+5)$$

Equations consist of two valid algebraic expressions, and a claim (the equals sign) that they are the same.

In this section, we will explore different functionalities that will help students get better at algebra, and means of handling their interaction.

Standard Operations

The standard algebraic operations are the foundation of algebra. They are the standard operators by which to modify equations or expressions. They provide the user assistance by automating the application of the operation to the object. The only realistic way to implement this is to manually type the operation in, since the operation involves operators (symbols), numbers and variables. For example:

$$2x^2+6x \quad \boxed{/2x} \Rightarrow$$

$$x+3$$

The tool can automate the evaluation of these operations for users at lower skill levels. At higher levels, the user can be challenged to complete the operation themselves. It is also applicable to equations.

$$\frac{2}{x} - 7 = 13 \quad \boxed{+7} \Rightarrow$$

$$\frac{2}{x} = 20 \quad \boxed{\times\, x/20} \Rightarrow$$

$$\frac{1}{10} = x$$

This is likely to be useful as an assistant feature. It will help identify students that understand the strategies required to solve problems, but that make mistakes when computing solutions (conceptual vs. procedural).

Grab Tool

Typing is not an ideal form of interaction, especially for users of lower skill. The addition of a grab tool allows the user to pick up and manipulate algebraic objects. It allows the transposition of terms in an equation.

$$\frac{2}{x}$$

$$\frac{2}{x} - 7 = 13 \quad \boxed{} \Rightarrow$$

It is important the grab tool only allows valid transpositions of this kind. Cross multiplication is often a stumbling block for students, if they have not grasped the basis of the rules. By limiting the user to only be able to make correct moves, users of low confidence will gain a more tangible sense of how the concepts work, since they are ultimately deciding what moves are attempted.

The grab tool also allows the user to grab expressions and drop them in a target area. This is a useful way to reduce typing and speed up operations.

$$x^2 \quad \boxed{x^2} \Rightarrow$$

Factorise Tool

Factorisation is an important skill for students to learn in high school algebra. The main challenges of factorisation are:

1. Identifying the factorisable units
2. Correctly executing the factorisation

The factorise tool, therefore, allows the user to select a range within a polynomial (equation or expression) to factorise. At lower skill levels, it auto-factorises the selection if any basic identities are detected. This isolates the conceptual skill of identification

$$3x^3 - x^2 - 3x + 1 \quad \Rightarrow$$

$$x^2(3x-1) - 3x + 1$$

At higher levels, the user must input the factors of their selection themselves, and the platform will verify the output.

Verification techniques like this are to be initiated once the user has shown a basic understanding of the concepts. The inputs the users make are then used to evaluate their arithmetic and procedural skill.

Expand/Collapse Tool

The inverse of factorisation is expansion, or multiplication of algebraic identities. This can be especially difficult for students, owing to the large number of terms that can emerge and a high potential for numerical error. Despite the mathematical operation being known as expansion, it is described as a “collapse” tool as it collapses a product of factors into a composite polynomial. It uses the same input structure as the factorisation, requiring the user to select a range within an expression on which to search for algebraic multiplication The collapse tool allows the user to select a range within a polynomial whose factors to collapse. At lower levels, it automatically renders the product of equations selected.

$$(x-3)(x-5)(x-7) \quad \Rightarrow$$

$$x^3 - 15x^2 + 71x - 105$$

As with factorisation, the higher-level user must input the product output themselves, and the program verifies correct expansion. It is important to note that neither feature suggests a transition from automated solving to fully typing out the solutions themselves.

Term Splitter Tool

There is sometimes a requirement to split like terms in a polynomial. Several procedures in the algebra syllabus explicitly involve identifying appropriate separations of expressions to allow them to be broken down into more manageable chunks. It is therefore necessary to include a term splitter tool that helps to teach the student how the technique is applied in the context of their work.

The term splitter tool allows a user to select a term in a polynomial, and split it into two terms of the same variable degree. The user can use a sliding scale, or type in the values manually. The tool will ensure that whatever is added to one side will be subtracted from the other.

$$x^2 + 5x + 6 \quad \Rightarrow$$

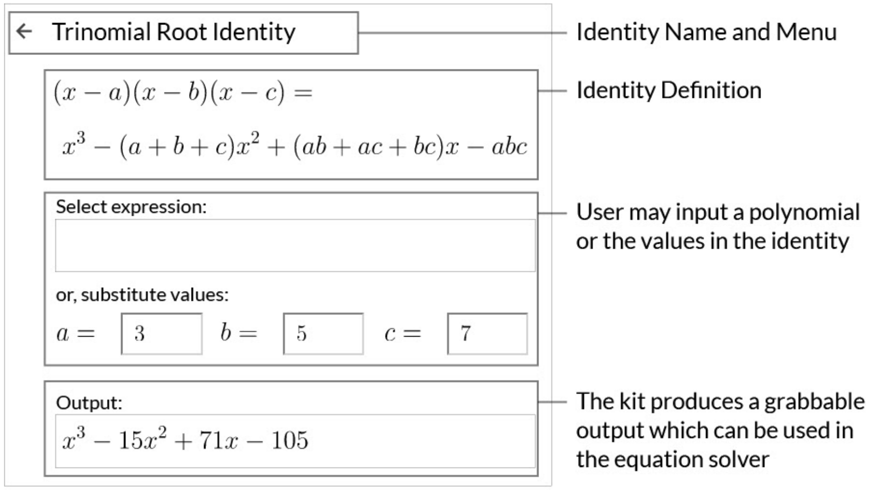
$$x^2 + \boxed{[5]x + [0]x} + 6$$

Theorem Kit

The students of Class 9 and Class 10 are required to learn several *identities.* These are results that they are expected to know, and can often provide quick solutions to questions.

The theorem kit offers a database of relevant results which they can use. It can be used on the basis of either substitution or parameters.

In the substitutional mode, a chosen expression may be placed into the theorem kit using the grab tool to produce a result. In the parametric mode, the user must input the chosen expression by identifying the correspondence of parameters. The mode used depends on which theorem is being used.



The first approach was taken along the lines of approach in the textbook (pictured left bottom). However, this was discarded as the interaction was not consistent with the primary mode of interaction so far. Therefore, a modified approach that more closely matches the rules of interaction so far discussed was created (pictured below).

$$\frac{x^3 + 3x^2 + 3x + 1}{x+1} \quad \Rightarrow \text{Term Splitter}$$

$$\frac{(x^3 + x^2) + (2x^2 + 3x + 1)}{x+1} \quad \Rightarrow \text{Factorise}$$

$$x^2\left(\frac{x+1}{x+1}\right) + \frac{(2x^2 + 3x + 1)}{x+1} \quad \Rightarrow \text{Collapse, split}$$

$$x^2 + \frac{(2x^2 + 2x) + (x+1)}{x+1} \quad \Rightarrow \text{Factorise, factorise}$$

$$x^2 + 2x\left(\frac{x+1}{x+1}\right) + 1\left(\frac{x+1}{x+1}\right) \quad \Rightarrow \text{Collapse, collapse}$$

$$x^2 + 2x + 1$$

Division Module

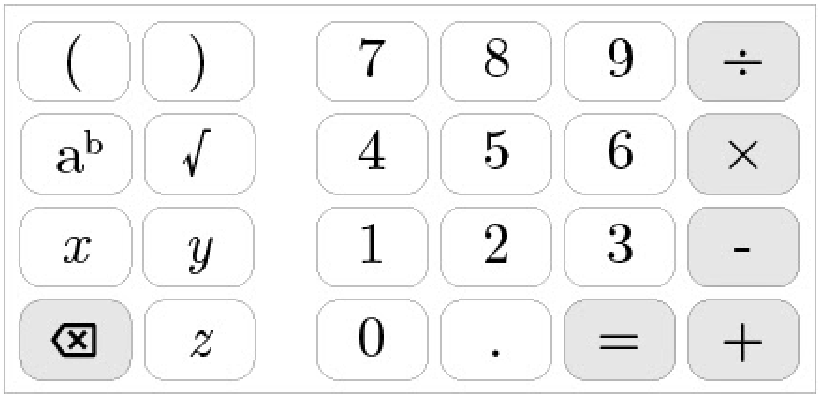
We will also examine a special case of the collapse function, when the selection includes a polynomial division. In this case, the user has to input quotient values that eliminate the highest term of the dividend. This "module" can be extended to apply to the remainder theorem questions that are also part of the curriculum.

$$\begin{array}{r|l}
 & x^2 + 2x + 1 \\ \hline
x + 1 & x^3 + 3x^2 + 3x + 1 \\
 & -\ (x^3 + x^2) \\ \hline
 & \quad 2x^2 + 3x + 1 \\
 & -\ (2x^2 + 2x) \\ \hline
 & \qquad\qquad x + 1 \\
 & \qquad -\ (x + 1) \\ \hline
 & \qquad\qquad\quad 0
\end{array}$$

Input

The tool requires a fairly significant amount of typing, especially for high-skilled users. Moreover, the typing is all mathematical, which makes a conventional keyboard somewhat inconvenient for this kind of application.

A specialised keyboard was therefore designed, with only the required keys available. This makes typing mathematical expressions in the program significantly more convenient.



Final Concept

We have now selected the algebraic methods required by the syllabus and explored the means of interaction that will be used to implement them. We have also discussed some additional features that are likely to be useful in the problem solving process. We will now discuss how these disparate methods for interacting with algebra functions are integrated into a cohesive concept.

System Structure

We have so far discussed only the methods and interactions of the algebraic functions needed for this concept, and the prototype will also largely focus on those issues, as they are the primary design contribution of this project.

That being said, the algebra training tool is a chosen intervention of this project to address its initial larger goals of improving learner outcomes in the e-learning environment. We will therefore look at how this tool is supposed to function within the educational environment, and how it aims to add value.

There are three parts to the system: the (virtual) classroom, the training tool, and the homes (of students and teachers). The simplified challenge of this project is that the teacher can no longer see the student in the classroom, and must therefore leverage both the tool and the time at home to substitute for the reduction in information about their students.

The users of this tool are grouped by classroom under a teacher, which falls under a larger grouping of users within the same school. The teacher or school administration identifies which grade each classroom belongs to, and the tool assigns an appropriate question bank to the users in the class group. Individual students, or the tool itself may add questions from outside the default grade level in response to their performance. The teacher may choose to set weekly performance targets (homework), either in terms of numbers of hours spent or number of problems solved, to ensure the students use the tool.

Each student's profile is tracked for their performance along different axes (arithmetic, conceptual, procedural), and these reports are shared with the teachers, along with a consolidated report on the class as a whole.

Based on this data, teachers can make time to address individual students who are struggling, identify areas where large numbers of students are struggling, and take corrective action. This leads to a shift from the classroom being the sole centre of learning, to being a special kind of learning interaction where the teacher knows ahead of time what their students need to learn.

This could make class hours more efficient and allow for fewer contact hours in general, which would help children that find it difficult to pay attention to their screens for 6-8 hours a day.

System Structure

Online Classes

Teachers use the data available to them on an individual and classroom level to better utilise limited contact hours.

Teachers	Students
• Better understand the learning progress of their class' • Less dependent on class hours as student practice time • Can make targeted interventions on common issues	• Better informed of their own learning in context of the curriculum • Can use class to solidify concept understanding • Less dependent on class hours to practice effectively (especially weaker students)

Training Platform

The platform creates a community group for each class, managed by a teacher.

Teachers	Students
• Sets class grade level • Sets weekly practice targets • Can view individual and class-level performance / progress reports • Can recommend focus areas to individual students	• Practice problems at their own skill level • Can use either the default problem set, or from the teacher's recommended topics

Home Environment

The platform also allows information transfer between teachers and students outside the class.

Teachers	Students
• Can use the reports to better plan classes • Can more accurately track student self-study • Tool assists diagnosis and identification of weaker students for additional help	• Tool enables increased transparency in home study • Practice is more reliable since the program flags any errors, which helps avoid making a habit of them

Difficulty Progression

A major value proposition of this tool is its ability to dynamically adjust both the types of questions served to users, and the tools available to solve them.

The harmonica is a popular musical instrument that is often used to teach children music. This is because there are two kinds of harmonica. Chromatic harmonicas are instruments like any other, and can be difficult to learn. However, most children's harmonicas are called diatonic harmonicas, which means they are constrained to only produce notes in a chosen musical scale.

The children being constrained to play "correct" music (in the harmonica's scale) helps them get a better understanding of what the scale is, through repeated exposure. It is similar to the function of training wheels on a bicycle.

This project attempts to apply those concepts to mathematical learning, and is also indicated by Vygotsky's research in particular. The tool's assistance features provide the "scaffolding" that allows students to expand their current set of capabilities.

The appropriate difficulty for each question is calculated based on three different kinds of skills values.

1. **Arithmetic skills**

Arithmetic Progression	
BODMAS	Integers
Cross-multiplication	Rational
Exponents & Roots	Irrational
HCF/LCM	Mixed

This is the arithmetic progression of the tool.
Consider a question that involves a polynomial of the form
$p(x) = \{a\}x^4 + \{b\}x^2 + \{c\}$,
Where a, b, and c are coefficients.
As the user's arithmetic skill (which is a value independent of the type of question) increases, the coefficients can take more available values, and more arithmetic techniques are required to solve them.
Easy: Integer coefficients, BODMAS
Medium: Rational coefficients and cross-multi
Hard: Irrational coefficients, exponents and roots
Hardest: Mixed coefficients and HCF/LCM

2. **Conceptual skills**
These skill values are defined for each question type. Each question type has its own unique skill tags attached, and question types are therefore also referred to as skills. Where a type has multiple children, the parent concept skill tag is shared among children, while procedural/arithmetic tags differ. Eg:

Quadratic polynomial *($ax^2 + bx + c$)*	Sum of Roots
	Product of Roots
	Solution of Quadratic Equation by Splitting the Middle

Consider the question of quadratic polynomials in Class 10. Understanding the sum and product of roots is a simple equation the student must learn based on a proof, hence is conceptual. While these questions would be a part of the training tool concept, they do not involve interaction with algebraic objects and are therefore not strictly relevant to this project. Existing tools like Brilliant.org achieve these outcomes very well.

Solving an equation by “splitting the middle” is a procedure a student must learn, based on their understanding of the parent concepts. The conceptual skill of the student in quadratic polynomials (as a parent of the question) is raised by correctly solving “splitting the middle” questions.

As discussed earlier, those questions will start with simple integer numbers that resolve cleanly, to isolate arithmetic skill from a clear understanding of the concept. As the user’s skill is demonstrated clearly, the questions and the coefficients associated get more difficult.

3. **Procedural skills**
These skills are unique to each question type. First, the user must solve questions where the arithmetic is heavily assisted by the assistant tools described previously and demonstrated later. As the user demonstrates conceptual command, assistance is withdrawn, the numbers get more complex, and the skill value rises further if the user can handle it.

Difficulty Progression

Skill Types

Difficulty

Arithmetic (universal)

- Auto-evaluate all arithmetic queries
- Occasional prompts for user-input
- Auto-evaluate only procedural tools
- Full user input of arithmetic functions
- Increase in co-efficient complexity
- Incorporation of BODMAS, cross-multiplication, exponents, HCF/LCM

Conceptual (sub-topic)

- Single-step introductory questions (objective, visuals, recognition: out of scope
- Fully assisted procedural questions
- Unassisted procedural questions
- Dependent concepts (repeats for a new, dependent concept after procedural threshold established)

Procedural (question type)

- Simple values, auto-arithmetic and all procedural tools available
- Arithmetic assistance withdrawn (making procedural tools only verifiers)
- Procedural assistance withdrawn (manual input in solver or on paper, and submission of the solutions

Ability Phases

The platform uses three phases of skill attainment to evaluate the skill level of its users.

Foundational

New question types are introduced to the user at this stage. A minimum viable degree of arithmetic ability, a threshold understanding of all pre-requisite concepts (from the dependency map), and threshold procedural ability in pre-requisite types characterise this phase.

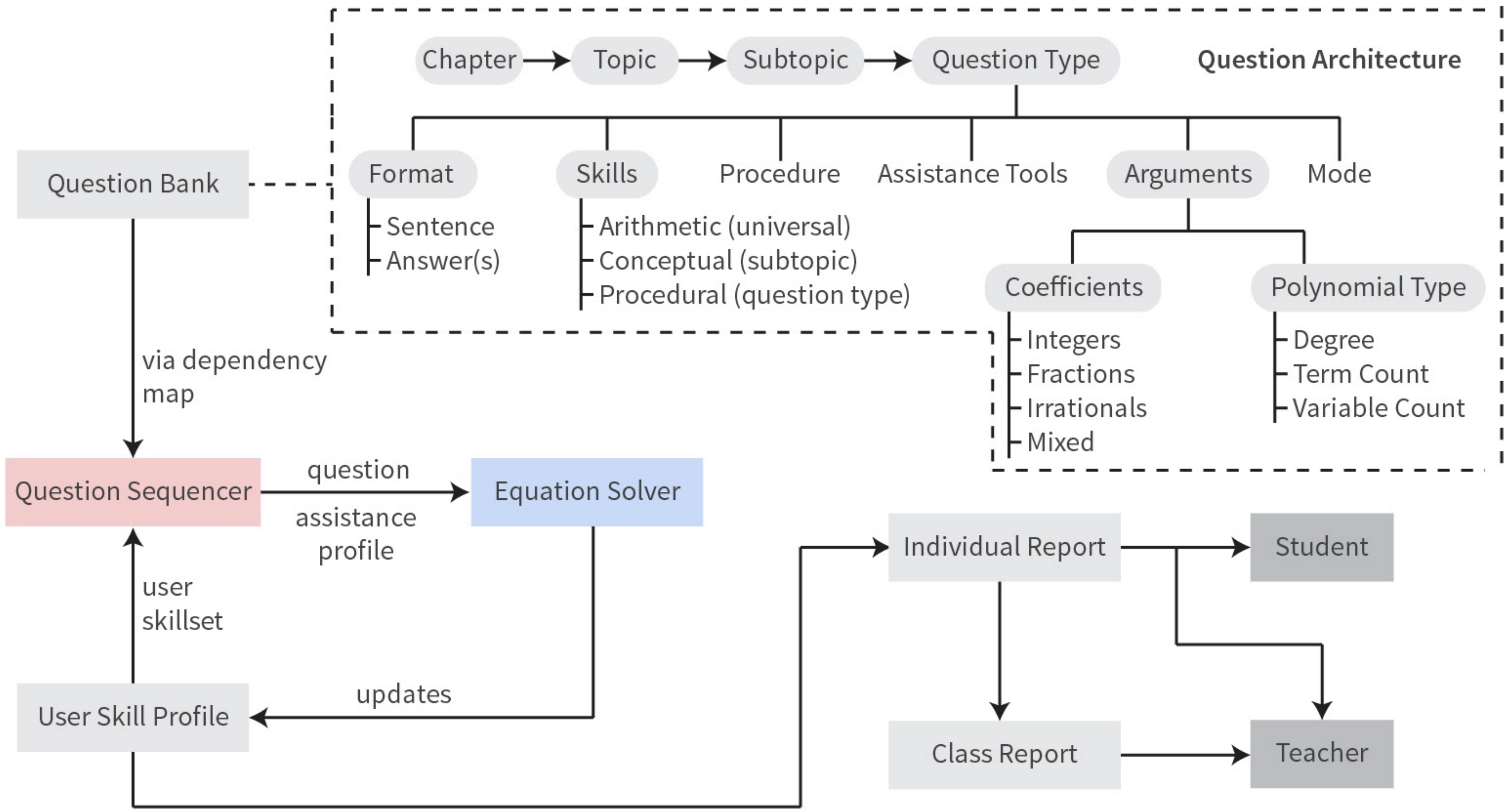
Conceptual

This phase is attained once the student is reliably able to solve problems in the type that are framed with relatively simple arithmetic, and making use of the procedural assistance tools.

Procedural

This phase is attained when a student can reliably solve problems in a type without the procedural assistance tools. It is not directly a function of arithmetic skill, though this is not achieveable without a fair amount of it.

Data Architecture



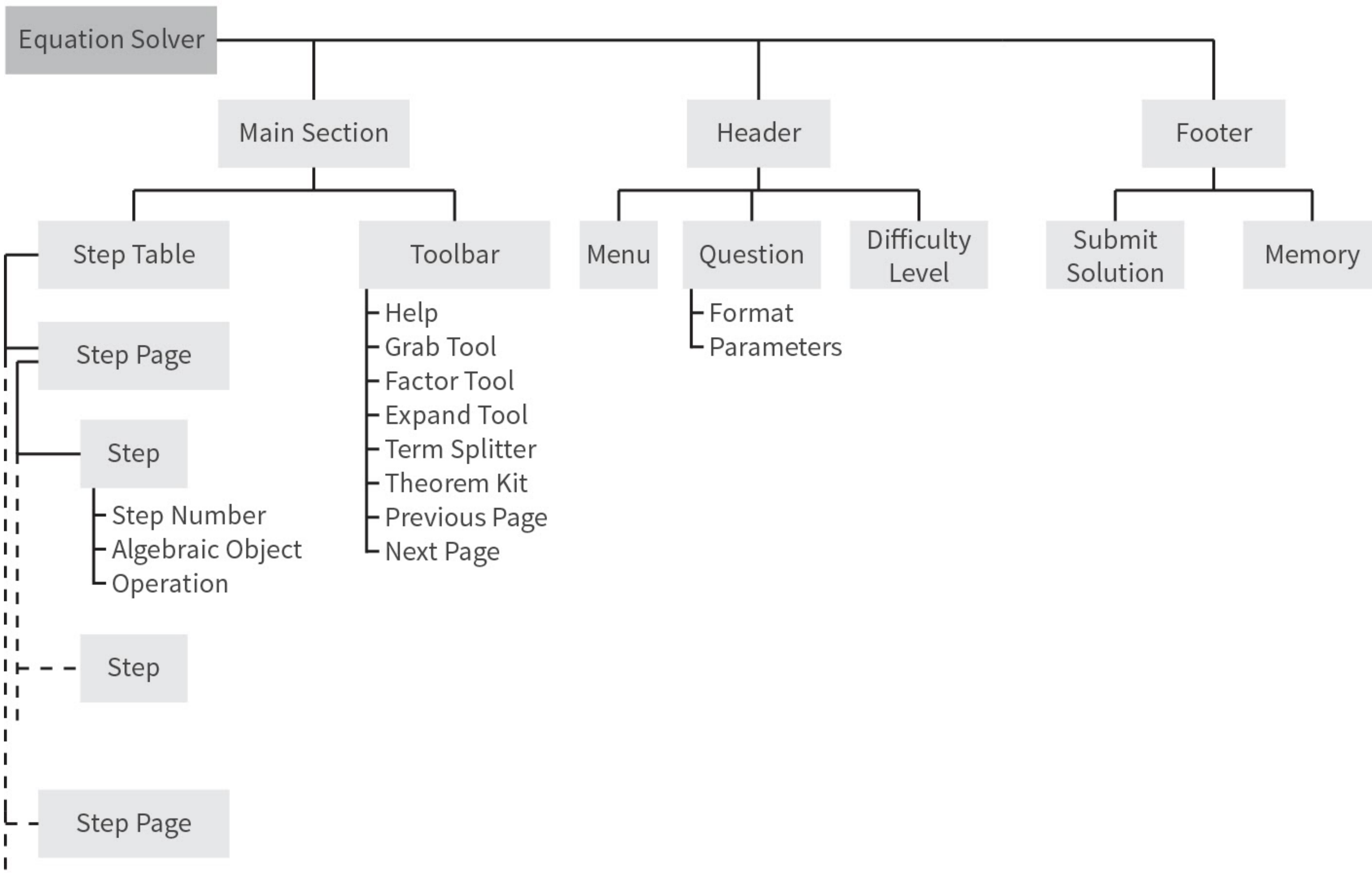
Prototype

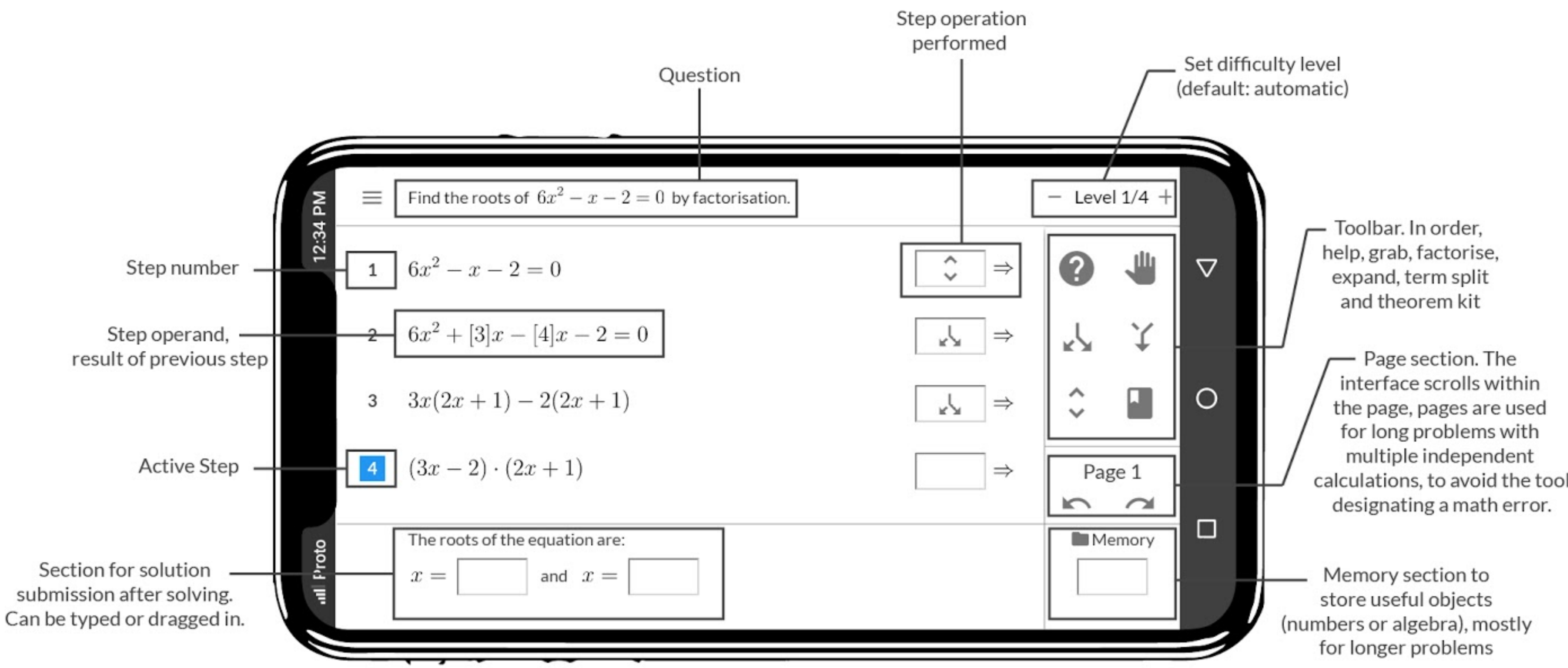
The project requires a prototype as proof-of-concept of the applicability of the interactions outlined above to the curriculum of the chosen context.

As mentioned previously, the full concept has several different modes of questions, many of which are borrowed from existing platforms. As these are not a part of the direct design contributions of this project, a prototype of the entire solution is out of scope.

The prototype will therefore examine the primary mode of interaction with procedural algebra, which is referred to as the equation solver.

Information Architecture




Question
Step operation performed
Set difficulty level (default: automatic)
Step number
Step operand, result of previous step
Active Step
Section for solution submission after solving. Can be typed or dragged in.
Toolbar. In order, help, grab, factorise, expand, term split and theorem kit
Page section. The interface scrolls within the page, pages are used for long problems with multiple independent calculations, to avoid the tool designating a math error.
Memory section to store useful objects (numbers or algebra), mostly for longer problems
12:34 PM
Find the roots of $6x^2 - x - 2 = 0$ by factorisation.
− Level 1/4 +
1 $6x^2 - x - 2 = 0$
2 $6x^2 + [3]x - [4]x - 2 = 0$
3 $3x(2x+1) - 2(2x+1)$
4 $(3x-2)\cdot(2x+1)$
Page 1
Memory
The roots of the equation are:
$x =$ and $x =$
Proto

Evaluation

Ideally, evaluation for this project would revolve around user testing. However, this is difficult to do, in absence of a working prototype. To address this, I created walkthroughs of user flows that displayed the functionality of the platform. These flows are difficult to display in a static format (as the prototype is a single dynamic page), so they may be viewed in my video presentation.

I sent these flows to teachers I had spoken to during the research phase of this project for their comments and feedback. I also evaluated the project on the basis of the solution requirements initially laid out.

Evaluation Results

Based on solution requirements:

- The solution maintains value even in the absence of much direct teacher input, which is a situation that may arise when teachers are not very comfortable with technology
- The solution is able to adapt to users' changing skill levels, and can distinguish between procedural, conceptual, and arithmetic skills
- Engagement appeal of the tool is unclear: this would require real-world testing with students. It may be that the tool's automation features make it less tedious for students to engage in, and if it is effective as a teaching aid, it could increase general interest in the subject and therefore, engagement. Conversely, the students may find doing mathematics practice on their phones more tedious than with pen and paper. Additionally, the solution is somewhat lacking in terms of "fun" elements, being targeted more as a classroom assistance tool than a more general learning platform.

From expert feedback:

- The teachers generally appreciated the ability of the tool to observe students' approach, and said that such a tool could potentially be very helpful to them
- They also appreciated the use of auto-complete features, saying they expected those to help students with low confidence practice math with more of a sense of personal agency, and improve their skills in the process.
- The teachers were somewhat unsure about the use of a phone or electronic device as a primary means of practicing math: suggesting the students had a chance of getting distracted while using the devices, and that they should not lose connect with pen and paper, since they need to use that medium in school
- The use of icons was somewhat confusing for them, although they understood why those icons were chosen once they were told what each function did.

Summary:
The core concept of procedural assessment appears to be strong, but a more engaging design and further fleshing out of the supplementary features would add greatly to the solution's value.

Conclusions

In this interaction design project, I studied the impact of the switch to e-learning in the wake of COVID-19 on secondary school mathematics teachers and students in Vellore, Tamil Nadu. I identified that a major concern of both teachers and students is reduction in information transfer, especially from students to teachers. I set on an automated assessment tool as an intervention to address this gap, arguing that the scalability of automation could be combined with more data-rich interactions with the questions, including working through the steps of questions, to produce a highly efficient and effective assessment tool.

Drawing on existing research in mathematics learning and assessment, as well as studying existing platforms that provide similar functions, I identified a lacuna in the market for an assessment tool that allowed for any interaction when dealing with algebra problems. I have suggested a design that draws on some of the established good ideas of existing platforms, while providing a novel means of interacting with algebraic objects that allows for procedural assessment in algebra. The design also takes care to separate components of the user's mathematical ability to enable better informed judgements from their performance.

The design was partially prototyped and evaluated on the basis of expert feedback from teachers, as well based on the solution requirements. While the concept of the solution appears to be strong, it would benefit from more attention to the engagement and comprehensibility of the platform than exists in its current form, which focuses on the proof-of-concept of procedural interaction through a digital medium, and its applicability to a large, previously unaddressed subject area like algebra.

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